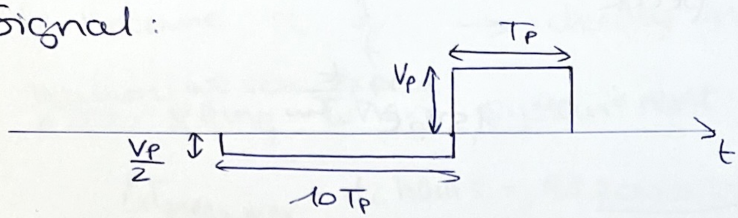
11 · 4 · 2024

①

Signal:



$T_P = 200us$

Preamp

$\sqrt{S_V} = 5nV/\sqrt{Hz}$

$\frac{1}{f^2}$, $f_c = 10MHz$

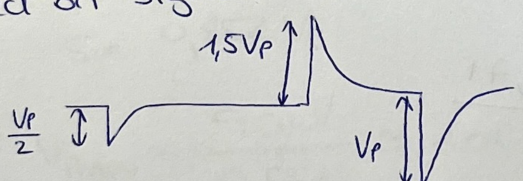
$f_{PA} = 500MHz$

a) optimum filter : whitening filter + matched filter

- whitening filter : CR HPF $\quad f_{HW} = f_c$

$$\tau_W = \frac{1}{2\pi\, 10^7 Hz} = 0{,}16 \cdot 10^{-7} = 16us \ll T_P$$

effect on signal of HPF:



- Matched filter: same shape of signal

$$b(t) \begin{cases} -\frac{1}{2} e^{-\frac{t}{\tau_W}} & 0 \le t < 10T_P \\ 1{,}5\, e^{-\frac{t-10T_P}{\tau_W}} & 10T_P \le t < 11T_P \\ -1\, e^{-\frac{t-11T_P}{\tau_W}} & t \ge 11T_P \end{cases}$$

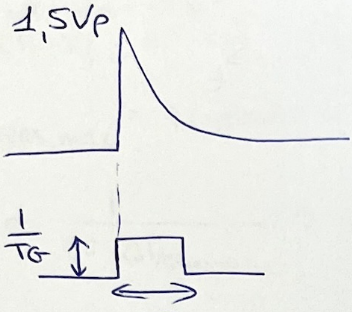
$$\left(\frac{S}{N}\right)_{OPT} = \frac{V_P}{\sqrt{\frac{S_V}{2}}} \cdot \sqrt{k_{bb}(0)}$$

Practical filter: CR-HPF + 1GI

- GI applied after the whitening filter on the highest peak

$1{,}5V_P$

$1{,}5V_P e^{-\frac{t}{\tau_w}}$



$\frac{1}{T_G}$

T_G

$$S = 1{,}5V_P \cdot \frac{1}{T_G} \tau_w \left(1 - e^{-\frac{T_G}{\tau_w}}\right)$$

$$N = \sqrt{S_v \cdot \frac{1}{2T_G}}$$

$$\frac{S}{N} = \frac{1{,}5V_P \frac{\tau_w}{T_G} \left(1 - e^{-\frac{T_G}{\tau_w}}\right)}{\sqrt{S_v \cdot \frac{1}{2T_G}}}$$

$$T_G = 1{,}25\,\tau_w$$

$$\frac{S}{N} = \frac{1{,}5V_P \cdot 0{,}8 \cdot 0{,}71}{\sqrt{S_v \frac{1}{2.5\,\tau_w}}}$$

$$V_{PMIN} = \sqrt{S_v} \sqrt{\frac{1}{40ns}} \frac{1}{1{,}5 \cdot 0{,}8 \cdot 0{,}71} = 5nV \frac{1}{2} \cdot 10^4 \cdot 1{,}17$$

$$\simeq 29{,}2\mu V$$

(2)

b) keeping the whitening filter makes $\frac{B}{f^3}$ component to become $\propto \frac{1}{f}$ → ideally infinite noise with CR-HPF + GI

However we can expect a zero setting when system is reset

$$\Delta T_{meas,max} = 12 \text{ hours} = 43200\ s$$

$$f_{i,eq} \simeq \frac{1}{2\pi\, \Delta T_{meas,max}} \simeq 3{,}7\,\mu Hz$$

same white noise of point a)

$$\sigma_w = \sqrt{S_v \cdot \frac{1}{2.5\,\tau_w}} = \frac{5nV}{\sqrt{Hz}}\sqrt{\frac{1}{40ns}} = 25\mu V$$

$$\sigma_{1/f} \simeq \sqrt{S_v f_c \ln\left(\frac{\frac{1}{40ns}}{3.7\mu Hz}\right)} = 5nV \cdot 10^3 \cdot \sqrt{10 \ln\left(\frac{10^{15}}{148}\right)} \simeq 86\ \mu V$$

$$\sigma_{TOT} \simeq 90\mu V$$

same signal of point a)

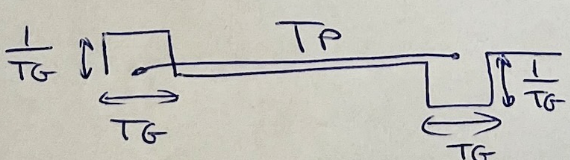
$$S = 0{,}852\, V_P$$

$$V_{PMIN} = \frac{90\mu V}{0{,}852} \simeq 106\mu V$$

dominated by $\frac{1}{f}$ noise

- CDF on two highest signal peaks

two identical integration windows for the sake of simplicity



$$\frac{1}{2\pi T_P} = 0{,}8MHz$$

$$\frac{1}{2T_G} = 25MHz$$

$$S = 1{,}5 V_P \cdot \frac{\tau_\omega}{T_G}\left(1 - e^{-\frac{T_G}{\tau_\omega}}\right) + V_P \frac{\tau_\omega}{T_G}\left(1 - e^{-\frac{T_G}{\tau_\omega}}\right) =$$

$$= 2{,}5 V_P \cdot 0{,}8 \cdot 0{,}71 = 1{,}42 V_P$$

$$\sigma_W \simeq \sqrt{2}\, \sigma_{WGI} \simeq 35\,\mu V$$

(fast computation $\frac{1}{2T_G} \gg \frac{1}{\pi \tau_i}$)

$$\sigma_{1/f} = \sqrt{S_{1/f} \ln\left(\frac{25}{0.8}\right)} \simeq 29\,\mu V$$

$$\sigma_{TOT} \cong 45{,}5\,\mu V$$

$$V_{P\,MIN} = \frac{45{,}5\,\mu V}{1{,}42} \simeq 32\,\mu V$$

c) see theory