09/01/2025 – solution draft

Pb1)

1) With a single GI available to filter the signal, the only limitation to 1/f noise is given by the zero setting of the acquisition instrument (e.g. an oscilloscope). Considering a worst case scenario of Tmeas=5h, the equivalent high pass filtering action is as low as $f_{HPF,eq} = \frac{1}{2\pi 5*60*60s} = 8{,}8\mu Hz$. For the optimization of the integration window we can consider only the wideband noise and then evaluate the contribution of 1/f noise with the chosen sizing of parameters.
   We consider a GI centered around the positive peak of the sinusoid with duration Tg-2x, where x is comprised between 0 and T/4. The resulting signal to wideband noise is
   $$\frac{S}{N_W} = \frac{\frac{2A}{\pi}\cos\left(\frac{2\pi x}{T}\right)}{\sqrt{\frac{2S_V}{T}\left(\frac{1}{2} - \frac{2x}{T}\right)}}$$
   By using a calculator it can be found that the maximum of this function is obtained for x=0.06T.
   For a simpler estimation of the obtainable result, x=0 can be used corresponding to Tg=T/2.
   With this sizing, the contribution of 1/f noise can be computed considering an equivalent LPF with a pole at 1/2TG=1/T=100kHz. Considering a GI with amplitude 1/Tg and duration Tg=T/2 we obtain the following SNR:
   $$\frac{S}{N_{TOT}} = \frac{\frac{2A}{\pi}}{\sqrt{\frac{S_V}{T} + S_V f_C \ln\left(\frac{\frac{1}{2T_G}}{f_{HPF,eq}}\right)}}$$
   The minimum amplitude that can be measured is Amin=1,1μV.

2) 1/f noise can be eliminated by using a high pass filter (i.e. a CR network) with a pole at 10kHz (1 decade above the corner frequency and below the signal). After this filter only white noise is present and an ideal matched filter can be theoretically designed, featuring a weighting function equal to a truncated sinusoid lasting for 1 period. In this case the optimum theoretical SNR is:
   $$\frac{S}{N} = \frac{A}{\sqrt{\frac{S_V}{2}}}\sqrt{\frac{T}{2}}$$
   Leading to Amin=632nV.

3) See theory.

Pb2)

1) The background current can be eliminated with a zero setting but its associated noise contribution must be considered. Assuming that all background is at the same frequency of the signal (efficiency of the PMT=10%), I_B=32pA.
   Since the signal preserves its shape after the preamplifier, a gated integrator can be used to collect it. Dark current noise and electronics noise are negligible with respect to background noise, thus leading to the following SNR:

$$\frac{S}{N} = \frac{I_P}{\sqrt{\frac{2q(I_P + I_B)F}{2T_G}}}$$

With a desider SNR=3 the minimum current that can be measured is $I_{P,min} = 244pA$, corresponding to $P_{P,min} = 6.1nW$.
In this case the background and signal shot noise are comparable.

2) When the bit rate changes, the duration of the integration window is changed accordingly.
With a bit rate of 10Mbit/s, $I_{P,min}$ *is reduced to* $39{,}2pA$, corresponding to $P_{P,min} = 980pW$ and the contribution of background and signal noise are still comparable.
On the other hand, with a bit rate of 1Gbit/s, $I_{P,min} = 2.2nA$, corresponding to $P_{P,min} = 55nW$ with the dominant noise contribution given by the signal shot noise.

3) See theory.