11/1/21 – Short solution

Problem 1

1) The minimum detectable signal with the optimum filter (weighting function with the same shape of the signal) is: $V_{p,MIN}$= $\sqrt{3S_V/(4T_P)} = 1{,}55\mu V$.
   A practical suitable filter is the gated integrator. Optimum duration is $T_G = \frac{4}{3}T_P$. $V_{p,MIN}$= $\sqrt{\frac{S_V}{2}\frac{1}{\sqrt{4/3\,T_P}*2/3}} = 1{,}64\mu V$
2) With a periodical signal, the SNR can be improved using a Boxcar. The maximum duration of the Boxcar weighting function must be 1s. The improvement factor is the same for the two cases of point 1) and it is equal to $\sqrt{2*\left[\frac{1s}{5}*\frac{T_W}{T_R}\right]*\frac{1}{T_W}} = \sqrt{4000} = 63{,}2$.
   (Note: T_w is the duration of the weighting function as in point 1) and T_R is the inverse of the repetition rate of the signal)
   Optimum filter+BI: $V_{p,MIN}$= $= \frac{1{,}26\mu V}{63{,}2} = 19{,}94nV$
   Optimized GI+BI: $V_{p,MIN}$= $= \frac{1{,}64\mu V}{63{,}2} = 25{,}95nV$
3) Both filters discussed in previous points only apply a low pass filtering action on both signal and noise. Therefore, an additional high pass filter is needed to limit 1/f noise.
   Exploiting all the signal-free interval between pulses to collect only noise, noise doubling is avoided. In this scenario, we compare the contribution of white noise to the one of 1/f noise. Triangular weighting function, with amplitude 1 and duration 2Tp:
   - $\sigma_W = \sqrt{\frac{S_V}{2}*\,T_P} = 6{,}32\mu V$
   - $\sigma_{1/f} \approx \sqrt{S_v f_c \ln\left(\frac{1/T_P}{1/2\pi(T_p+9T_p)}\right)} = 0{,}364\mu V$

   Comment: with the selected filtering option, 1/f noise contribution is negligible with respect to white noise. Therefore, the minimum detectable signal is the same of point 1 and 2. Same considerations hold when a rectangular filtering is applied on the single pulse.

Problem 2

1) A suitable APD for this application features R=0.2, w_N (neutral region depth) = 200nm, w_D (depleted region depth) = 5µm, G=100, F=2, dark current = 20fA. As a result, PDE=0.65, S_D=0.262A/W. The signal can be filtered with a LPF, f_P=1kHz. The major noise contribution is due to the electronics (amplifier and input resistor). The minimum APD current signal that can detected with SNR=1 is 63.8pA, which corresponds to $P_{INC,MIN}$= 2.4pW.
2) Noise would be ideally infinite (practically extremely high) in presence of 1/f noise since no high pass filtering action has been introduced so far. With a zero-setting, in the worst case scenario (acquisition after 2hours), $I_{APD,MIN}$ = 198pA, $P_{INC,MIN}$= 7.56pW.
3) The laser could be modulated on-off, either electrically or by using a mechanical chopper in front of the laser. [Note: a sinusoidal modulation could also be exploited, but the laser has typically a lower power in this mode]. A squarewave +/- 1 LIA is used to demodulate the signal. In this way, the DC component of both signal and noise is removed.
   After modulation (well above 2kHz to avoid 1/f noise), demodulation and LPF we are in the same scenario of point 1, except for signal power that has been halved.
   Therefore $P_{INC,MIN}$= 1.2pW.