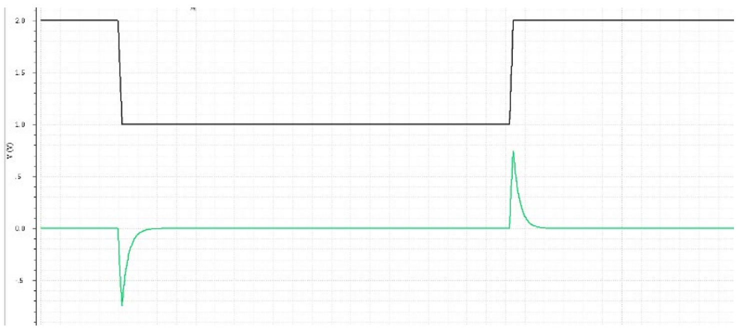
Problem 1

A) The noise superimposed to the signal includes a $1/f^2$ component; therefore, it is possible and necessary to use a whitening filter before a matched filter. The whitening filter is a high pass filter with unitary gain at high frequency and a pole at $f_W=f_C$=20kHz. The filter acts also on the signal: it removes the constant background and produces two exponential signals with amplitude |Vp| and time constant Tw=1/(2π*20KHz)=7.96μs as sketched in figure.



The optimum filter is the cascade of the whitening filter and the matched filter, which has a weighting function made of two exponential shapes as the filtered signal reported above (green).

SNR_{OPT}=Vp/sqrt(S_v/2)*sqrt(T_W).

The minimum measurable signal is Vp_{MIN}~1μV

B) We can use a passive LPF after the whitening filter. The best choice for the filter parameters is $T_F=T_W$. We can measure a single exponential and the best choice is to acquire it at a measurement time equal to T_W after the peak of the exponential signal itself.

SNR=V_P/e / sqrt(S_V*1/(4T_W))

With this solution the minimum measurable signal is Vp_{MIN}~1.92μV

C) We can use a digital filtering scheme after the whitening filter. The noise at the input of the sampling scheme would be correlated because of the preamplifier with Tn=1/(2π150MHz) = 1.06ns

Therefore, to collect uncorrelated noise samples we could use a sampling frequency f_S=1/(5Tn)=188MHz.

The digital filter can collect both the exponential signals resulting from the whitening filter action on the input pulse and apply an exponential weighting function that resembles the signal shape.

SNR=sqrt(2)*Vp/sqrt(Sv) * sqrt(Tw/2.5)

With this solution the minimum measurable signal is Vp_{MIN}~1.585μV

D) See theory.

Problem 2

1) We can use one strain gauge to detect the signal and one strain gauge as dummy cell to compensate for temperature variations. The best configuration is a Wheatstone bridge with a reference branch consisting of two constant value resistors and two strain Gages on the other branch. The bias voltage is 3.3V (constant) and the signal is readout by a differential amplifier. The input signal of the preamp is $vd=V_{PS} / 4 * G * \varepsilon$, where ε is a sinusoidal signal. We can use a strain gage with gage factor=2.
   A deformation of 1µstrain corresponds to vd=3.3V/4 * G * 1 µstrain = 1.65 µV.

2) We want to design a system that can properly acquire a sinusoidal signal whose frequency can be between 83.3Hz (=5000rpm) and 166.6Hz (=10000rpm). We need a LPF and a HPF to limit the noise. To avoid signal loss fp_{HPF} = 1/10*83.3Hz = 8.33Hz and fp_{LPF}=10*166.6Hz = 1.66KHz.
   With these two filters, the overall noise contribution is 680nV leading to a SNR = 2.42 with a signal of 1 µstrain.

3) We can modulate the signal to avoid that it mixes with 1/f noise component of preamp noise. To do so we can switch the bias between $+V_{PS}$ and $-V_{PS}$ (square wave modulation) with a switching frequency higher than fc. After the demodulation we have a sinusoidal frequency at the motor rotating frequency. If we use a LPF at 1.66KHz directly after the square wave demodulation we can obtain a SNR=6.44 with a signal of 1 µstrain. As an alternative, we can use a second demodulation at the motor rotating frequency: in this case the output signal is a DC signal. We can use a LPF for example at 10Hz obtaining a SNR=59 with a signal of 1 µstrain.

4) See theory.