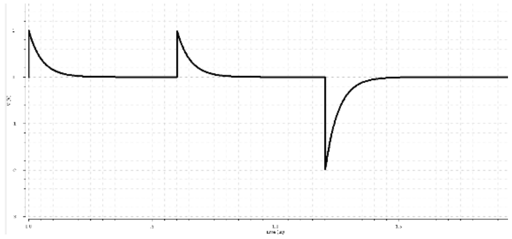
Problem 1

A) Without any filtering, noise is divergent since the $1/f^2$ component is not limited at low frequency. Therefore, $V_{P,MIN}$ that can be measured is infinite.

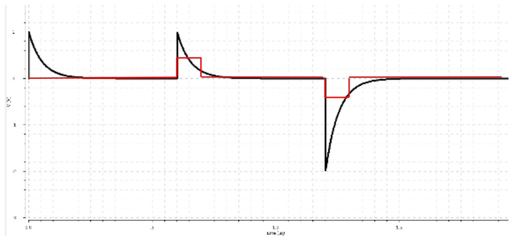
B) The ideal filter consists of two stages: a whitening filter followed by a matched filter. The whitening filter is a high-pass filter featuring a pole at f_C=3MHz. The filter acts not only on noise (which is white at its output), but also on the signal. The shape of the signal is modified: at the output of the whitening noise we have a sequence of three exponential pulses, two with positive amplitude V_P followed by a negative pulse with amplitude $2V_P$. The time constant τ is 1/(2*pi* f_C)=53ns.
A sketch of the signal waveform (amplitude vs time) is sketched in the following figure.



The optimum filter features a weighting function having the same shape of the signal. In this scenario, $V_{P,MIN}$=17.7µV.

C) If we could use a single integration window, the best choice is to collect the signal where it has the highest amplitude (in module), that is at the beginning of the negative peak. It can be demonstrated that the best duration of the integration window in this case is 1.25τ=1.25*53ns=66.25ns. In this case, $V_{P,MIN}$=24.18µV.

D) To deal with the $1/f^3$ component, the best solution is exploiting the same whitening filter of point b). As a result, we have a white and a 1/f component. We need an additional high-pass filter to limit 1/f noise at low frequency. We could exploit one main feature of the signal, that is it has both a positive and a negative pulse (see the figure above): if we use a CDF, we can collect more signal and cancel-out low-frequency components of correlated noise. We will now use a gated integrator and two integration windows (see next figure) having the same amplitude ($1/T_G$) and the same width (equal to 1.25τ for simplicity).

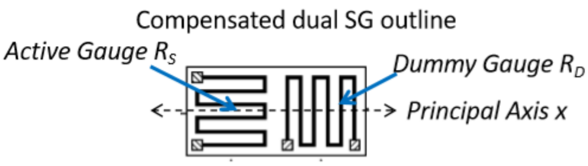


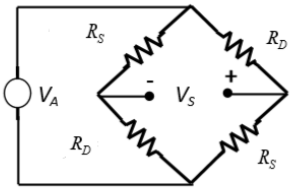
$S=2V_P*1/T_G*\tau*0.71*(1.5)=1.7V_P$
(the last term is due to the fact that we are summing two pulses but the amplitude of the positive one is half of the negative pulse amplitude).
$N_W= \text{sqrt}(2* 1/2T_G*S_V)=38.8\mu V$
$N_{1/f}\approx \text{sqrt}\{2*S_V f_c*\ln[(1/2T_G)/1/(2\pi T_P)]\}=44.8\mu V$
$N_{TOT}\approx 59.3\mu V$
$V_{P,MIN}=34.88\mu V$.

Problem 2

A) The best solution consists in using two pairs of orthogonal strain gauges in a Wheatstone-bridge configuration. As shown in the figure, we have two active and two dummy gauges. V_A=5V.

See slides for more details.





B) Signal in microstrain:

10N/(3x6mm^2)= 22*10^4 N/ mm^2 * ε. As a result, ε=1.26μstrain.

If we consider a strain gauge with G=2, we have V_S= ½ V_A*G* ε = 6.3μV.

To preserve the signal spectrum, we can use a preamplifier with a bandwidth limitation 10 times higher than the signal bandwidth, i.e. f_{PA}=1kHz. To have a SNR≥10 we need:

$\sqrt{[S_V \frac{\pi}{2} f_{PA}} \le \frac{6.3}{10}\,\mu V$. Thus, $S_V^{1/2}$<16nV/Hz$^{1/2}$.

C) With a low-pass filtered zero setting, we can avoid noise doubling. Therefore, we can consider the same noise generators of point B) and compare their contribution with 1/f noise contribution.

Zero setting once an hour results into a high-pass filtering action with an equivalent lower-bandwidth limitation at ≈1/(2π3600s) = 44.2μHz. The upper-bandwidth limitation is given by the preamplifier. Therefore

$$\sqrt{[S_V f_C \ln(\frac{1kHz}{44.2\mu Hz})} \ll 0.63\mu V \rightarrow f_C \ll 92Hz$$

To relax the constraint on f_C we could consider an amplifier with wideband noise lower than the maximum value computed in point B.

D) With a square-wave modulation we can make the contribution of 1/f noise negligible no matter what is its corner frequency. Obviously, the lower the corner frequency, the lower can be the modulation frequency. For example, we can select an amplifier with f_C = 1kHz and modulate at 10kHz. To avoid the presence of a DC component of the modulating wave, we need a square-wave bias for the Wheatstone bridge with zero-DC value, e.g. 5V/-5V. In this scenario, signal and wideband noise contribution after demodulation are the same as before, V_S = 6.3μV and same noise.