

Problem 1

- A) Without any filtering, noise is only limited by the preamp. $V_{p_{MIN}} = \sqrt{S_V \frac{\pi}{2} f_{PA}} = 125 \mu V$
 Gated Integrator: best solution is to start the integration window at the beginning of the signal.
 Considering a GI with amplitude $A=1$ and duration $0 < T_G < T_P$, the expression for signal and noise is:
 $S = V_P(T_G - T_G^2/2T_P)$
 $N^2 = S_V/2 * T_G$
 The best SNR is obtained for $T_G = 2/3 T_P$, leading to a $V_{p_{MIN}} = 4.1 \mu V$
- B) Without any filter, $1/f$ noise contribution would lead to an infinite noise.
 One possibility is to add an initial zero setting with a duration much higher than the GI to avoid noise doubling. Equivalent high-pass bandwidth (worst case) = $1/2\pi \times 8 \text{ hours} = 5.5 \mu \text{Hz}$.
 White noise is the same of point A) (we use the same GI), $\sigma_W = 2.7 \mu V$ while $\sigma_{1/f} = 4.8 \mu V$. Therefore $\sigma_{Tot} = 5.5 \mu V$. As an alternative, a baseline restorer could be exploited.
 Please note that a **HPF could not be used since repetition rate of pulses is random**.
- C) Now we know the distance between pulses. We could use a CDF to avoid signal loss.
 We can have an integration window for noise larger than the window on signal to limit its impact.
 With an integration window for noise of $40 \mu s$, $\sigma_W = 1.1 * 2.7 \mu V = 2.97 \mu V$ and $\sigma_{1/f} = 1.43 \mu V$.
 Therefore $\sigma_{Tot} = 3.3 \mu V$
- D) We can now use a Boxcar to improve SNR. We need a weighting function that lasts for $1s$.
 Therefore $\tau_{BOXCAR} = 1s/5 * T_G/T_R = 1s/5 + 6.67 \mu s/50 \mu s = 26.68 ms$. The improvement factor with respect to the single CDF is $\sqrt{2 * \tau_{BOXCAR}/T_G} = 89.4$. $\sigma_{Tot} = 3.3 \mu V/89.4 = 36.9 nV$.

Problem 2

- A) Since the same kind of detector must be used for different wavelengths, we need a very thin neutral region to avoid signal loss at short wavelengths and a thick depleted region to have a high efficiency at long wavelengths. Some reasonable parameters could be:
 $R = 0.2$
 $t_N = 100 nm$
 $t_D = 30 \mu m$
 with these data $PDE(400nm) = 30\%$, $PDE(500nm) = 72\%$, $PDE(800nm) = 76\%$
 noise is due to the preamplifier and the feedback resistor.
 $(S_{i,IN,TOT})^{1/2} = 1.62 pA/(Hz)^{1/2}$
 $\sigma_{Tot} = 20 nA$ considering the limitation given by the preamp.
 Worst case for radiant sensitivity is $400 nm$
 $S = 0.096 A/W$
 $P_{MIN} = 207 nW @ 400 nm$
 (for comparison $P_{MIN} = 40.8 nW @ 800 nm$)
- B) $1/f$ noise of the preamp prevents the measurement of a continuous signal. Modulation is necessary. If the light source does not provide this operation mode, we can use a chopper.
 For example, we can use 50% duty cycle with ON phase $\ll 1s$, e.g. $50 ms$
 By using a CDF on the single pulse $\sigma_{Tot} = 48 pA$. $P_{MIN} = 500 pW @ 400 nm$.
 Another possible solution to recover the modulated signal would be a Lock-in Amplifier.
- C) PMTs have an internal gain. This feature could make the electronics noise negligible.
 Typical values for a PMT are $G = 10^6$, $F = 2$, $n_D = 1000 e^-/s$, $PDE(400nm) = 20\%$, $PDE(800nm) = 0.5\%$.
 The contributions of signal and dark counts noise ($2qI_s G^2 F$ and $2qI_D G^2 F$) sum up with read-out electronics noise. The dominant contribution is the noise associated to the signal itself.

$I_{S,MIN} = 100\text{pA}$ considering only white noise and the bandwidth limitation given by the preamplifier. The worst case with PMTs is the signal at 800nm because of the low PDE. Therefore, $P_{MIN} = 31\text{nW}$ @800nm.

D) See theory for a detailed description of PMTs and SPADs.

In photon counting, the minimum signal that can be measured is limited by dark counts and background. In this case we can assume that there is no background. A reasonable value for dark count rate of a SPAD is 100cps. In a time window of 5ns, the number of dark count events is $N_D = 100\text{ cps} * 5\text{ns} = 500 * 10^{-9}$. Therefore, $N_{S,MIN} = 1$ absorbed photon. Similar considerations hold for a PMT leading to the same result.