Problem 1

a) The $1/f^2$ noise component coming with the signal would give a divergent contribution if no action is taken. Nevertheless, in a practical case a limitation is given, e.g. with a zero setting of the measurement instrument. With a zero setting performed 30s before the measurement of the signal of interest, for example, the contribution of the 1/f2 noise component gives a $\sigma \approx 2.8mV$. In this case the contribution of white noise is negligible. Therefore, the minimum detectable amplitude of the signal is 2.8mV.

b) The presence of $1/f^2$ noise component makes it necessary to use a whitening filter with a pole at the corner frequency. The time constant of the High Pass whitening filter is 15.9µs, much shorter than the duration of the signal. As a result, the shape of the signal is modified by the whitening filter resulting into two exponential signals (a positive one and a negative one) with an exponential decay time constant equal to the one of the whitening filter. The optimum filter must have the same shape of the signal; the minimum theoretical signal that can be detected with this filter is as low as 9.4µV.

c) a gated integrator can be used to practically filter the signal. In particular, we can use two integration windows, one on each exponential signal resulting from the whitening filter, and subtract them. The effect of this filtering on the signal is a low pass filtering action given by the duration of the integration window, and a high pass filtering action that is given by the subtraction of two integration windows, resulting into an equivalent pole of 1/(2π150µs).

- When the sinusoidal disturb is negligible, we can use an integration window as long as 1.25τ to maximize the signal to noise ratio on the single exponential signal. In this case the minimum detectable signal is as low as 10.46µV.

- When dealing with a significant sinusoidal disturb, we can modify the duration of the integration window to completely filter out the disturb. In particular, in this case we can choose an integration window of <u>25µs</u> on each exponential signal obtaining a full cancellation of the sinusoidal disturb and a minimum detectable amplitude of 10.54µV. Please note that this choice is slightly worse in terms of signal to noise ratio but it is crucial to filter out the disturb.

d) if we consider the case with the disturb, we can keep an integration window of 25µs. The use of two integration windows that are subtracted to double the signal is also effective to provide a high-pass filtering action on noise with an equivalent frequency of 1/(2π*150µs)=1.06kHz. Therefore, the contribution of 1/f noise is $\sqrt{2 * S_V f_C \ln\left(\frac{20}{1.06k}\right)} = 857nV$

Problem 2

1) See theory
2) See theory
3) The presence of both wideband and 1/f noise makes it necessary to use both a low pass and a high pass filter. The pole of the low pass filter can be chosen as high as 1kHz to avoid any signal loss (i.e.the pole is one decade higher than the signal bandwith), while a high pass filtering action can be performed with a zero setting before the measurement giving rise to a high pass filtering action with a pole at 1/(2π*1h)= 1/(2π*3600s)=44.2µHz. The contribution of 1/f noise is as high as 1.127µV; if we use a single strain gage in a Wheatstone bridge, with a continuous bias of 100mV and a gage factor of 2 the minimum detectable signal is 20µstrain. With a temperature coefficient of $4*10^{-3}$ /K, a temperature variation of 0.01 degrees gives rise to a signal variation comparable with the minimum detectable signal. Therefore, the maximum temperature variation that can be tolerated is at least one decade lower than the minimum signal, i.e. 1mK.
4) It is possible to improve the measurement using two strain gages and a modulation and demodulation scheme. The best option is using a square-wave modulation and demodulation. In this case we only have the contribution of white noise limited by a LPF that can be designed with a pole at 1KHz. In this case we have a a $\sigma \approx 198nV$ corresponding to a minimum detectable signal of 1.98µstrain.