PROBLEM 1

Current noise of the electronic circuitry connected to the PMT anode. It is negligible in comparison to the background current noise amplified by the PMT therefore it will not be considered

A1) The PMT shot noise can be considered white, since it has autocorrelation width negligible with respect to the signal duration. Consequently, the optimum weighting function is constant over the pulse duration. This optimum filtering is well approximated by a GI of duration $T_p = 100ns$ synchronized to the optical signal, therefore:

$$\frac{S}{N} = \frac{I_{es}}{\sqrt{(S_e + S_B)/2T_p}} = \frac{I_{es}}{\sqrt{q(I_{es} + I_B)/T_p}}$$

Minimum amplitude $I_{es,\min}$ corresponds to S/N=1

$$\frac{I_{es,\min}}{\sqrt{q(I_{es,\min} + I_B)/T_p}} = 1$$

__If__ $I_B \ll I_{es,\min}$ the background is negligible

$$I_{es,\min} = \frac{q}{T_p} = 1,6 \cdot 10^{-12} A = 1,6 pA$$

We can verify that

$I_{e,\min 1} = 1,6 pA \gg I_B = 0,4 fA$

that is, in this case the approximation is valid.

A2)

The minimum signal corresponds to just one photon detected per pulse, hence to a photoelectron rate

$$n_{es,\min} = \frac{I_{es,\min}}{q} = \frac{1}{T_p} = 10^7 el/s$$

(which is indeed much higher than the background)

The minimum measurable amplitude of the optical pulse in terms of photon rate is

$$n_{ps,\min} = \frac{n_{es,\min}}{\eta} = 10^8 ph/s$$

B1) The measurements of the various pulses are uncorrelated, hence mean values and mean square deviations of the single pulses are added in the averaged measurement.

$$\frac{S}{N} = \frac{N_{m1} I_{e,m1}}{\sqrt{N_{m1} q(I_{e,m1} + I_B)/T_p}} = \frac{I_{e,m1}}{\sqrt{q(I_{e,m1} + I_B)/N_{m1} T_p}} =$$

Hence

$$\frac{I_{e,m1,\min}}{\sqrt{q\left(I_{e,m1,\min}+I_B\right)/N_{m1}T_p}}=1$$

<u>Assuming</u> $I_B \ll I_{e,m1,\min}$ the background is negligible

which leads to

$$I_{e,m1,\min}=\frac{q}{T_p N_{m1}}=1,6\cdot 10^{-14}A=16 fA$$

We can verify that

$I_{e,m1,\min}=16 fA \gg I_B = 0,4 fA$

that is, the approximation is valid also in this case, the photocurrent noise is still dominant.

The minimum signal corresponds to a photoelectron rate

$$n_{e,m1,\min}=\frac{I_{e,m1,\min}}{q}=\frac{1}{N_{m1}T_p}=10^5 el/s$$

which corresponds to a minimum amplitude of the optical pulse in terms of photon rate is

$$n_{p,m1,\min}=\frac{n_{e,m1,\min}}{\eta}=10^6 ph/s$$

B2)

In a measure averaged over a <u>moderate</u> number (around 100) of pulses the minimum measurable amplitude becomes progressively lower as N_{m1} increases, but it is still high enough to keep the photocurrent noise contribution dominant with respect to the background noise. Therefore, the <u>minimum measured amplitude decreases as the reciprocal of the number of pulses N_{m1}</u>

$$I_{e,m1,\min}=\frac{q}{T_p N_{m1}}=\frac{I_{es,\min}}{N_{m1}}=\frac{I_{es,\min}}{100}$$

C1) The measurements of the various pulses are uncorrelated, hence mean values and mean square deviations of the single pulses are added in the averaged measurement.

$$\frac{S}{N}=\frac{N_{m2}I_e}{\sqrt{N_{m2}q\left(I_e+I_B\right)/T_p}}=\frac{I_e}{\sqrt{q\left(I_e+I_B\right)/N_{m2}T_p}}=$$

Hence

$$\frac{I_{e,m2,\min}}{\sqrt{q\left(I_{e,m2,\min}+I_B\right)/N_{m2}T_p}}=1$$

Let us assume first that still it is $I_B \ll I_{e,m2,\min}$, i.e. that the background noise be still negligible with respect to the photocurrent noise. We get

$$\frac{I_{e,m2,\min}}{\sqrt{qI_{e,m2,\min}/N_{m2}T_p}} = 1$$

However, this leads to

$$I_{e,m2,\min} = \frac{q}{N_{m2}T_p} = \frac{I_{e,\min 1}}{N_{m2}} = 1,6\cdot 10^{-18}\,A = 1,6\ aA$$

which shows that the assumption is unfounded, the approximation of negligible background noise is not valid in this case

$I_B = 0,4\,fA \gg I_{e,m2,\min} = 1,6aA$

Let us see then if we can get a correct result with the opposite approximation, that is, by considering negligible the photocurrent noise with respect to background noise. We get

$$\frac{I_{e,m2,\min}}{\sqrt{qI_B/N_{m2}T_p}} = 1$$

Denoting by $N_{Bp} = n_B T_p = 10^{-3}$ the mean number of background electrons emitted per pulse, we get

$$I_{e,m2,\min} \approx \sqrt{\frac{qI_B}{N_{m2}T_p}} = \frac{q}{T_p}\sqrt{\frac{n_B T_p}{N_{m2}}} = \frac{q}{T_p}\sqrt{\frac{N_{Bp}}{N_{m2}}} = 25aA$$

in this case it is confirmed that the approximation of negligible photocurrent noise is valid

$I_{e,m2,\min} = 25aA \ll I_B = 400aA$

C2)

In a measure averaged over a high number (around 10^6) of pulses N_{m2} the minimum measured pulse amplitude becomes so low that the background noise contribution becomes dominant. This causes the minimum measured amplitude to decrease as the reciprocal of the square root of the number of pulses $\sqrt{N_{m2}}$

D)

In measurement of single pulses (see Section A) the baseline current I_B is not significant, hence it is not necessary to subtract it from the measured current. In fact, I_B is negligible even in comparison to the minimum measurable pulse current.

In measurements averaged over 100 pulses (see Section B) it is still not necessary to subtract the baseline I_B from the measured current, since it is much smaller than the minimum measurable pulse current.

In measurements averaged over 10^6 pulses (see Section C) the measurable pulse amplitude becomes so low that the baseline becomes significant and must be separately measured and subtracted.

The baseline can be measured by means of a GI equal to that employed for measuring the signal pulse and open in a time interval with equal duration, but after the optical pulse (or before it).

The uncorrelated noise brought by the subtracted baseline doubles the background noise in the pulse measurement.

PROBLEM 2

A)

With white noise the optimum weighting function is equal to the signal waveform. In this case it is well approximated by a Gated Integrator (GI) normalized to unit gain

At the GI output:

signal $s_y = V_P$ noise $\sqrt{\overline{n^2_{y,op}}} = \frac{\sqrt{S_{V,u}}}{\sqrt{2T_P}}$ $\left(\frac{S}{N}\right)_{op} = \frac{V_P}{\sqrt{S_{V,u}}}\sqrt{2T_P}$

minimum measurable $V_{P\min,op} = \sqrt{\overline{n^2_{y,op}}} = \frac{\sqrt{S_{V,u}}}{\sqrt{2T_P}} \approx 1,3\mu V$

Without filtering:

signal $s_y = V_P$ noise $\sqrt{\overline{n^2_x}} = \frac{\sqrt{S_{V,u}}}{\sqrt{4T_n}}$ $\left(\frac{S}{N}\right)_{x} = \frac{V_P}{\sqrt{S_{V,u}}}\sqrt{4T_n}$

the factor of improvement is

$$\sqrt{\frac{\overline{n^2_x}}{\overline{n^2_{y,op}}}} = \sqrt{\frac{T_P}{2T_n}} = \sqrt{2500} \approx 50$$

B)

The signal duration T_P is divided in N time intervals of duration T_s

$$N = \frac{T_P}{T_s}$$

The signal amplitude is sampled at the center of each interval T_s and the samples are summed weighted by a weight 1/N, in order to have filtering normalized to unit gain.

The factor of improvement is $\sqrt{N}$.

C)

The dependence of the result on the sampling frequency $f_s = 1/T_s$ is explicited by substituting in the equation $N = f_s T_P$

$$\left(\frac{S}{N}\right)_s = V_P \frac{\sqrt{4T_n}}{\sqrt{S_{V,u}}}\sqrt{f_s T_P} = V_P \frac{\sqrt{4T_n}}{\sqrt{S_{V,u}}}\sqrt{\frac{T_P}{T_s}}$$

However, this equation is valid only as long as the noise samples are uncorrelated.

See theory lessons: Low Pass Filter 3

D) since more pulse are now available and since these vary slowly it is possible to exploit the information using a box car as reported in lesson : Low Pass Filter 3