Problem 1

Signal: rectangular pulse with T_P = 500 μs duration V_P amplitude, to be measured	**Noise:** $\sqrt{S_{V,u}} = 40nV/\sqrt{Hz}$ (unilateral) white with noise band limit $f_n=2,5MHz$
$f_S = 1/T_S$ sampling frequency (selectable)	

The amplitude of the signal above specified must be measured in presence of the noise above reported.

A) Select a practical filter that well approximates the optimum filter for the measurement required. Calculate the corresponding S/N equation and evaluate the minimum measurable amplitude. Compare with the measurement done without filtering.

B) Consider now to operate with discrete-time filtering, taking samples of the input waveform at a frequency f_S and elaborating the sample values. Explain how a filtering of the same kind as that seen in (A) can be implemented with discrete-time filtering. Obtain the corresponding S/N equation and the minimum measurable pulse amplitude, showing how the result depends on the number of samples taken of the signal

C) Employing an approximate representation of the noise, describes in details the dependence of the result of the discrete-time filtering seen in (B) on the sampling frequency f_S and compare it with the result of the continuous-time filtering seen in (A).

D) Consider now that the amplitude of signal slowly changes on a few seconds timescale and the signals are received with a 1kHz fixed repetition rate. How can you use this information to improve the signal to noise ratio?

(NB: see text also on the other side of the sheet)

Problem 2

<table>
<tr><td colspan="2">RECTANGULAR OPTICAL PULSE
- $T_p = 100 ns$ duration
- $r_p = 100 kHz$ pulse repetition rate
- electrical signal synchronous to light pulse available
- variable amplitude</td></tr>
<tr><td>PHOTOMULTIPLIER TUBE
- Gain G=·10^6 , negligible Excess Noise F≈1
- Detection efficiency η=0,10
- Background at PMT cathode (dark current and environment light): $I_B = 0,4 fA$ current</td><td>CURRENT PREAMPLIFIER
- Noise (unilateral) at amplifier input
$\sqrt{S_{iA}} = 1\, pA/\sqrt{Hz}$</td></tr>
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For optical pulses with the features above specified, it is required to measure in various conditions the amplitude in terms of photon rate employing as analog photodetector the photomultiplier tube (PMT) and the preamplifier above specified.

A) Consider first to measure a single pulse. (1) Select a filter suitable for obtaining the best possible signal-to-noise ratio, explaining the reasons of your choice. Evaluate the minimum measurable photocurrent pulse amplitude, pointing out the noise components and their relative role, possibly finding out a dominant component (2) Evaluate in this condition the actual minimum measurable pulse in terms of photoelectrons and the corresponding minimum measurable photon rate.

B) Consider now to measure the amplitude by averaging over a moderate number of pulses, namely N_{m1}=100 pulses. (1) Evaluate the minimum measurable photocurrent pulse amplitude, pointing out the time taken by the measurement in this case and the relative role of the noise components, possibly finding out a dominant component. (2) Obtain the equation that shows how the minimum amplitude decreases as the moderate averaged number N_{m1} is increased

C) Consider now to measure the amplitude by averaging over a high number of pulses, namely $N_{m2}=10^6$ (one million) pulses. (1) Evaluate the minimum measurable photocurrent pulse amplitude attained, pointing out the time taken by the measurement in this case and the relative role of the noise components, possibly finding out a dominant component. (2) Obtain the equation that shows how the minimum amplitude decreases as the high averaged number N_{m2} is increased; in particular, evaluate the factor of the improvement obtained with respect to the measure of a single pulse.

D) The background current I_B is a baseline of finite value added to the measured photocurrent I_e . For each one of the three cases above considered, analyze quantitatively whether it is necessary or not to measure and subtract this baseline. In the cases where it turns out to be necessary to subtract the baseline, evaluate how the minimum measurable amplitude is changed by the baseline subtraction.