Problem 1

a) Without additional filter, white noise is limited only by the preamp. $V_{p,MIN}=\sqrt{S_V * \frac{\pi}{2} * f_{PA}} = 177\mu V$

The best possible SNR can be (theoretically) obtained with the matched filter, i.e. a filter having a weighting function with the same shape of the signal waveform. $V_{p,MIN}= 4.2\ \mu V$

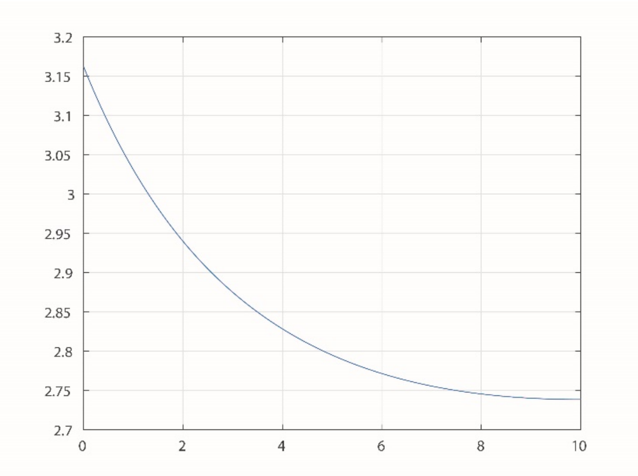
b) A suitable practical filter is the Gated integrator. Both the starting point and the duration of the integration window impact on the SNR. In order to optimize the SNR, it is necessary to collect the signal where it has its maximum height, while we have to find out if it is advantageous or not to collect the signal were its height is equal to ¼ of the peak amplitude. So, we have to consider an integration window <u>centered around the peak</u> and we have to maximize the SNR as a function of the duration T_G. Parameter of the GI: amplitude 1, duration T_G=10µs+2x with $0 \le x \le 10\mu s$.

$S=V_P(10\mu s+1/4*2x)$

$N^2=(S_V/2)*(10\mu s+2x)$

The maximum SNR is for x=0, that is T_G=10µs

For completeness the SNR waveform as a function of x, considering an integration window centered around the signal peak, is here reported. The x-axis is expressed in µs while the y-axis has an arbitrary unit.



With T_G=10µs centered around the peak, $V_{p,MIN}= 4.47\ \mu V$

c) Because of the preamplifier, the noise is not an ideal while noise but it has an exponential autocorrelation function with time constant $\tau=1/(2*\pi*f_{PA})$=3.18ns. we consider the noise correlated for ≈5 τ = 15.9ns. If we choose a Tsample higher than 15.9ns, then we can easily calculate the SNR, because noise samples are practically uncorrelated. We can choose Tsample=16ns.

$S=V_P(1/4*1/4*20\ \mu s/16ns +10\ \mu s/16ns)$

$N=\sqrt{(S_V * \frac{\pi}{2} * f_{PA}) * ((\frac{1}{4})^2 * \frac{20}{0.016} + \frac{10}{0.016})}$

With respect to the single sample (result of point a), $V_{p,MIN}$ is reduced by a factor equal to $\sqrt{N_{eq}}$, where Neq=1/16*20/0.016+10/0.016=703.

$V_{p,MIN}= 6.67\ \mu V$

By further increasing the sampling frequency, the result obtained with the optimum filter can be approached.

d) without any additional filter, the noise would diverge because of the 1/f component. A high-pass filtering action on the noise is required. Two possible solutions are:

1. High-pass with $\tau_{CR} \gg T_P$ to avoid distortion of the signal peak (we want to collect only the peak with a GI if we use the same solution of point b). we can choose τ_{CR}= 1ms.
   $N_{1/f}^2= S_V*f_C*\ln(f_{GI}/f_{CR})$= 3.39µV and white noise is the same of point b. $V_{p,MIN}= 5.6\ \mu V$

2. CDF with two gated integrators. We can use a GI with a duration much higher than the signal duration to sample the baseline, thus avoiding noise doubling. For example, we can use a GI with a duration of 300µs to sample the baseline, and a GI as the one used to solve point b for the signal. $N_{1/f}^2$ = 3 µV and white noise is the same of point b. $V_{p,MIN} = 5.38\ \mu V$

Problem 2

a) The repetition frequency is limited by the distance that we want to measure, which is approximately 200km. the light pulse has to travel this distance and come back. Maximum time required: 2*200km/30cm/ns= 1.33ms. $f_{LASER} \leq 1/1.33$ms = 0.75kHz.
A conservative choice could be f_{LASER}=700Hz.
At 500nm, L_0=1µm.
Main features of APD: detection efficiency η, Gain G, noise factor F.
reasonable parameters from theory:
G=100, F=2.5 and with reflectivity at surface R=0.2, neutral region at surface t_N=100nm and depleted region t_D=5µm, the detection efficiency can be computed: η=72% at 500nm.
Therefore, S= 0.29.

b) The RC-load time constant is 2ns, so the shape of the signal is preserved.
Noise contributions are shot noise of signal and dark current, noise of the preamp and noise of the resistor. The best filtering solution is a gated integrator with T_G=1µs.
The expression of the SNR considering all these contributions has to be written. Then, we can suppose that the electronics and resistor noise are the dominant contributions. In this case: $S=I_D*G*R$. N^2= (SV+SI*R^2+4kTR)*1/2T_G. $Id_{,MIN}$=30pA. Noise of electronics is in fact dominant with respect to shot noise of current. P_{MIN}= $Id_{,MIN}$/S = 103pW.

c) The repetitive nature of the signal within a time window of 1s can be exploited to improve the SNR by using a boxcar. The maximum duration of weighting function of the boxcar has to be 1s.
τ_{BOXCAR} = 1s/5 * 1µs * 700Hz = 140µs.
with respect to the single gated integrator (point b) the SNR is improved by a factor $\sqrt{2 * 140\mu s/1\mu s} = 16.7$. $Id_{,MIN}$=30pA/16.7=1.8pA. P_{MIN}=6pW.

d) During an observation window of duration = 1µs, the number of collected events due to dark counts is N_{DC}=10cps*1µs = 10^-5. N_S is the average number of photons of the signal.
S/N= $N_S/(N_S+N_{DC})$. SNR=1 when N_S=1, so with a SPAD we can distinguish and count a single photon.
The detection of a single photon in a time window of 1 µs means that we are able to detect a power as low as $P_{MIN}=E_{ph}$(at 500nm)/1 µs = 1.24/0.5*10^-19 J/1 µs = 400fW.