Problem 1.

A) It can be demonstrated that the maximum SNR is achieved with an integration window starting in sync with the signal and having a duration T_G=1.25τ. In this case $V_{P,MIN}$=7.04µV.
B) The best solution is sampling the background level integrating only the background for a time interval much longer than the one used for the signal. In this way, the SNR is not affected by the background subtraction.
C) Since the signal varies slowly over time, a Boxcar is the best solution. A conservative choice is T_F=12.5ms to have a weighting function that goes to almost zero in 1s (considering 5τ). The improvement in the SNR with respect to the single GI is $\sqrt{2 * T_F/T_G}$ = 44.7.
D) See theory.

Problem 2.

A) Additional filtering is necessary to limit 1/f noise. To this aim we can use a high pass filter with fi=1mHz. to further improve the measurement, also a low pass filter can be used with fs=10Hz.
Noise contribution=6uV, dominated by 1/f component.
Signal: 25mV/4 * α * ΔT. Typical value of α=4*10^-3.
ΔT_{MIN}=6uV/(25mV/4 * α) = 0.24°C.
B) The best solution is sampling the signal where it has the maximum amplitude (in absolute value). To limit 1/f noise we sample both when the signal is maximum (50 samples) and when the signal is minimum (50 samples) and then we subtract the two measurements. See slides of lesson SR17 (BPF3) from page 16 for more details.
S= 25mV/4 * α * ΔT = 25mV/4 * α * ΔT
$N^2 \approx S_v * 1/(2T_G) * f_{PA}/(2fs)$, where T_G=100/(2fs) and fs is the signal frequency (100KHz).
ΔT_{MIN}= 0.4°C.
C) The constraint on signal frequency and bandwidth prevent the use of a resonant filter.
Instead, a LIA can be used, with a LPF having a fp=10Hz.
S= 25mV/4 * α * ΔT
$N^2 = 2 * S_v * \pi/2 * fp$.
ΔT_{MIN}= 0.0045°C

D) A PN junction temperature sensor can be used in this case. See theory for more details.