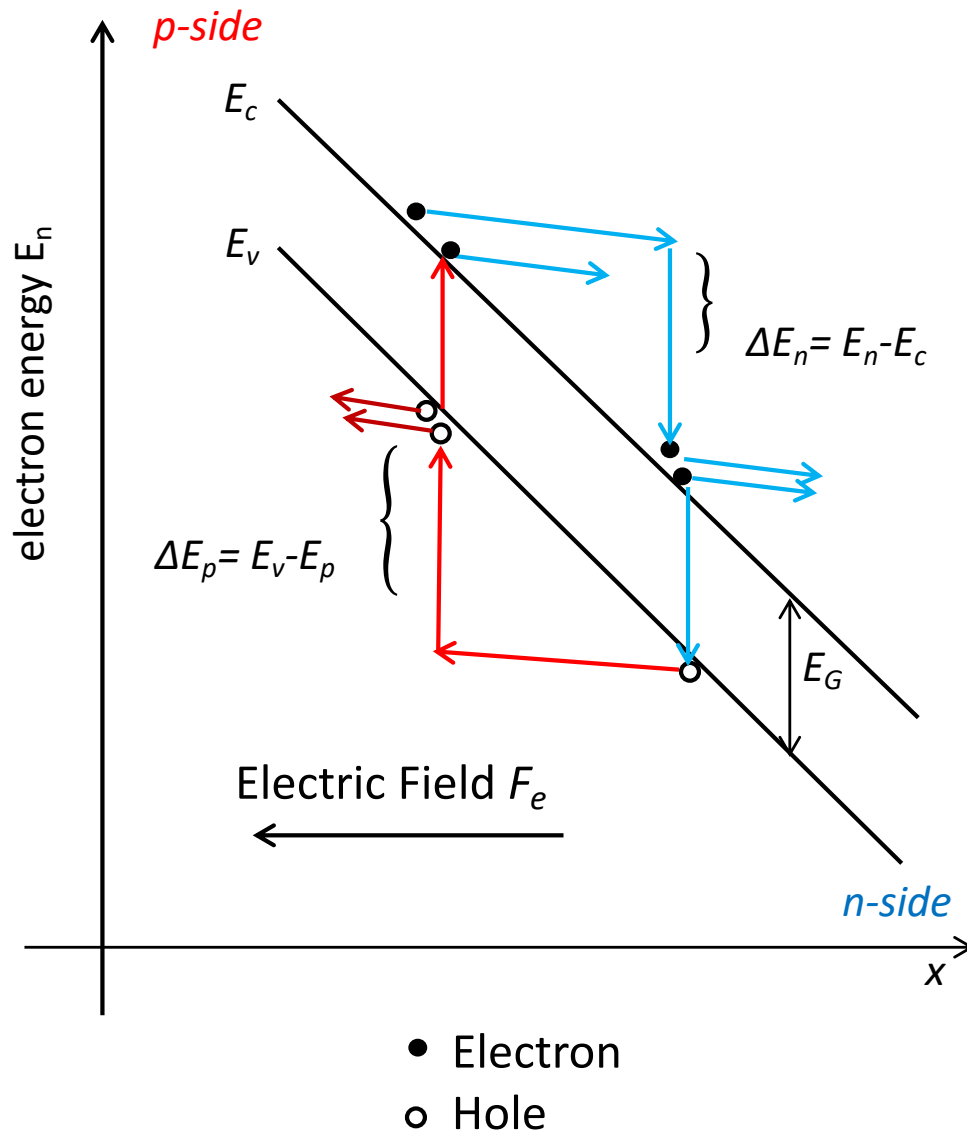


## COURSE OUTLINE

- Introduction
- Signals and Noise
- Filtering
- **Sensors: PD5 – Avalanche PhotoDiodes**

- Impact ionization in semiconductors
- Linear amplification by avalanche multiplication of free carriers
- Silicon Avalanche Photodiodes (Si-APD) and evolution of the device structure
- Statistical behavior of the avalanche multiplication and limits to the gain



- ❑ A free electron drifting in the field gains kinetic energy  $\Delta E_n = E_n - E_c$
- ❑ Part of  $\Delta E_n$  is transferred to lattice vibrations by scattering events
- ❑ Because of energy and momentum conservation, a ionizing collision can occur only when
 
$$\Delta E_n > 1,5E_G$$
- ❑ Until reaching such  $\Delta E_n$  the carrier travels without ionizing. The carrier multiplication thus has a dead-space; it is a **discontinuous statistical** process
- ❑ There is inherently a **positive feedback** loop in the process, because also holes can ionize by impact
- ❑ a cascade of ionizing collisions produces **avalanche multiplication** of carriers

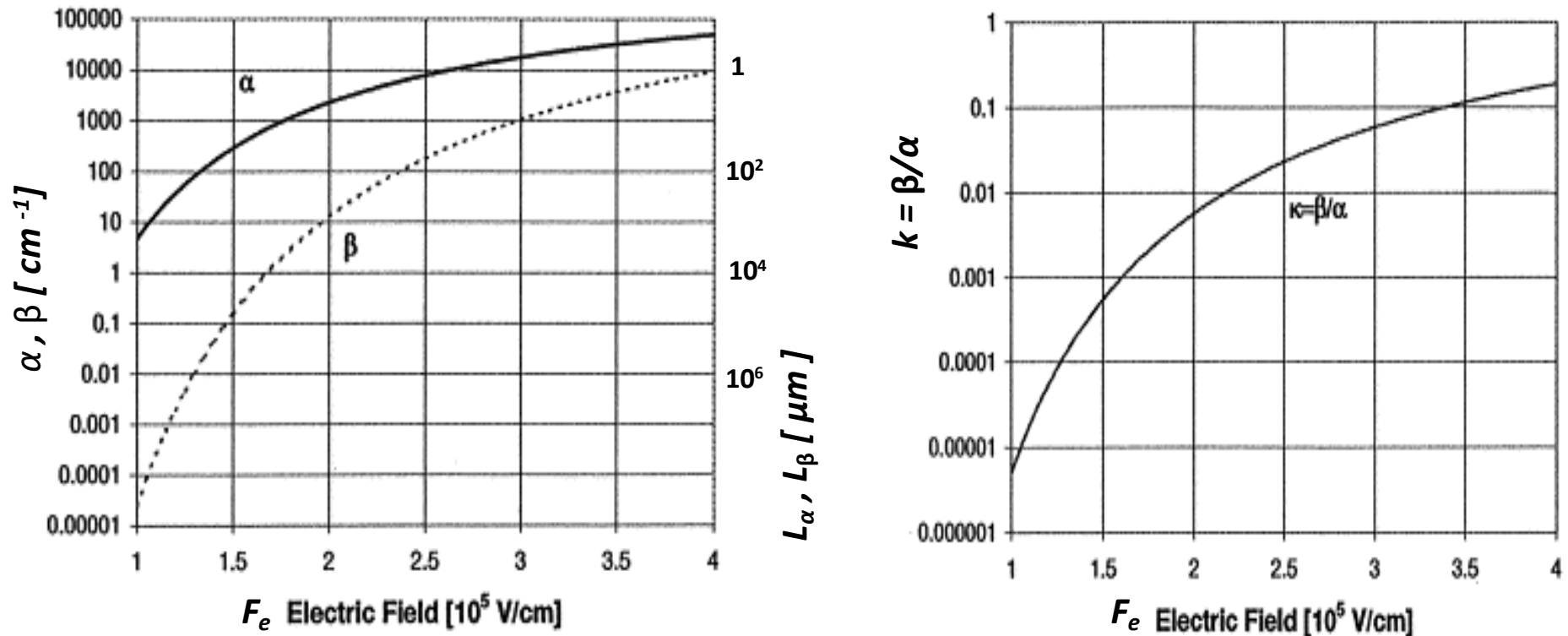
- The carrier multiplication can be analyzed with a **continuous statistical model**, based on the **average in space of the true discontinuous random process**.
- The continuous model provides a good approximation if the width of the multiplication region (high-field region) is definitely larger than the mean path between ionizing collisions. **The model is inadequate if the high-field region is very thin**, i.e. for width smaller than or comparable to the mean path between collisions.
- The model considers the probability of ionizing impact of a carrier as continuously distributed in space (i.e. it considers the average of many trials of carrier multiplication started by a primary charge).
- The **ionizing coefficients  $\alpha$  for electrons and  $\beta$  for holes** are defined as the probability density of ionization in the carrier path; that is, for a carrier traveling over  $dx$  the probability of producing impact-ionization in  $dx$  is

$$\alpha dx \text{ for electrons} \quad \text{and} \quad \beta dx \text{ for holes}$$

- The mean path between ionizing collisions thus is

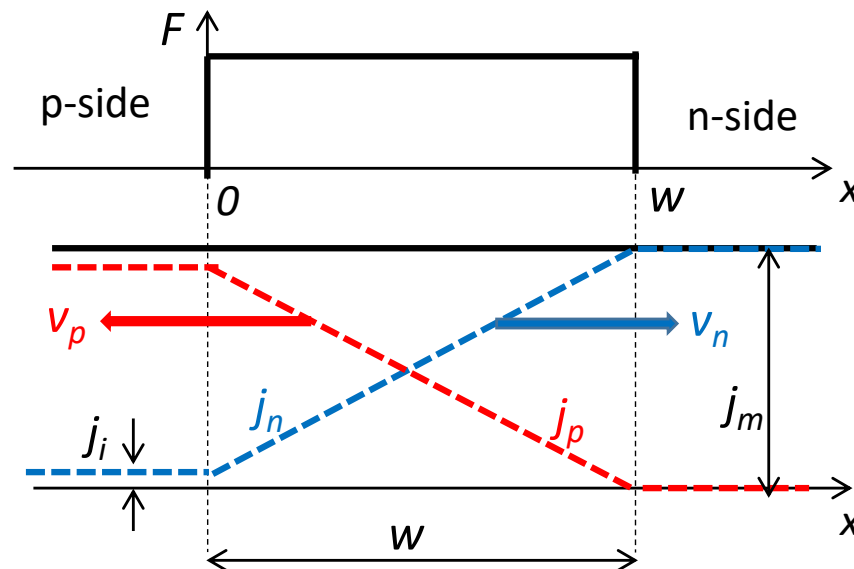
$$L_\alpha = 1 / \alpha \text{ for electrons} \quad \text{and} \quad L_\beta = 1 / \beta \text{ for holes}$$

- **The features of the multiplication process strongly depend on the relative intensity of the positive feedback, hence on the value of  $k = \beta/\alpha$** , which is different in different materials:  $k \ll 1$  in Silicon,  $k > 1$  in Ge and  $k \approx 1$  in GaAs and other III-V materials



- $\alpha$  and  $\beta$  **rapidly increase with the electric field  $F_e$** . They can be described with good approximation by  $\alpha = \alpha_0 \exp\left(-\frac{F_{no}}{F_e}\right)$  and  $\beta = \beta_0 \exp\left(-\frac{F_{po}}{F_e}\right)$   
 In Silicon  $\alpha_0 = 3,8 \cdot 10^6 \text{ cm}^{-1}$ ,  $F_{no} = 1,75 \cdot 10^6 \text{ V/cm}$ ;  $\beta_0 = 2,25 \cdot 10^7 \text{ cm}^{-1}$ ,  $F_{po} = 3,26 \cdot 10^6 \text{ V/cm}$
- $k$  is  $\approx 0,1$  at high electric field  $F_e$  and as  $F_e$  decreases  $k$  strongly decreases (because the dynamics of valence-band holes and conduction-band electrons are different)
- $\alpha$  and  $\beta$  **markedly decrease as temperature increases** (because stronger lattice vibrations drain more energy from carriers in the path between ionizing collisions)

- Even employing the continuous model, the complete mathematical analysis of the avalanche multiplication of carriers is quite complicated and will not be reported.
- However, the basic features of avalanche diodes can be clarified by analyzing a simple case. In a PIN junction with uniform and constant field higher than the impact ionization threshold, let us consider the stationary avalanche current due to the injection from the p-side of a small primary current of electrons  $j_i$



Note that:

1. e-h pairs are generated, hence there are **both** electron and hole currents, even in case the ionization by holes be negligible (i.e.  $\beta \approx 0$ )
2. The total current is constant  $j_m = j_n + j_p$
3. The p and n carriers of the avalanche form a **dipole-like mobile space-charge** (mostly p at p-side, mostly n at n-side) that adds a **field opposite to the junction field** (due to the fixed ion space charge)

In the simplest case  $\alpha = \beta$  (e.g. in GaAs) the equation is simply and we obtain:

$$j_m = \frac{j_i}{1 - \int_0^w \alpha(x) dx} = \frac{j_i}{1 - I_i}$$

$$I_i = \int_0^w \alpha(x) dx$$

is called **ionization integral** and has a clear physical meaning:

it is the probability for a carrier to have an ionizing collision in the path from  $x=0$  to  $x=w$

The current  $j_m$  is the primary current  $j_i$  amplified by the **multiplication factor M**

$$M = \frac{j_m}{j_i} = \frac{1}{1 - I_i}$$

In cases with  $\alpha \neq \beta$  the equation can still be integrated and the results can still be written in the form

$$j_m = \frac{j_i}{1 - I_i}$$

$$M = \frac{j_m}{j_i} = \frac{1}{1 - I_i}$$

but the ionization integral  $I_i$  is now the integral of an effective ionization coefficient  $\alpha_e$

$$\alpha_e = \alpha \exp \left[ - \int_0^w (\alpha - \beta) d\xi \right]$$

so that in this case

$$I_i = \int_0^w \alpha_e(x) dx = \int_0^w \alpha \exp \left[ - \int_0^w (\alpha - \beta) d\xi \right] dx$$



$$j_m = \frac{j_i}{1 - I_i}$$

$$M = \frac{j_m}{j_i} = \frac{1}{1 - I_i}$$

- The ionization integral  $I_i$  in any case **strongly** depends on the **applied bias voltage  $V_a$**  and on the **temperature T**
- $I_i$  is nil until the field  $F_e$  produced by  $V_a$  attains level sufficient for impact ionization
- Computations and experiments show that the **rise of M gets steeper as the high-field zone gets wider**. This is quite intuitive, since a wider zone corresponds to a higher number of collisions, which enhances the effect of the increased impact ionization probability due to an increase of the electric field

- When the applied bias voltage  $V_a$  reaches a characteristic value  $V_B$ , the Ionization Integral  $I_i \rightarrow 1$  and, according to the equation,  $M \rightarrow \infty$  and  $j_m \rightarrow \infty$
- $V_B$  is called **Breakdown Voltage**; it is a characteristic feature of the diode, ruled by the distribution of the electric field  $F_e$  and by the dependance of  $\alpha$  and  $\beta$  on the electric field  $F_e$  and on the temperature  $T$
- $V_B$  increases with the **temperature  $T$** . The increase is different in devices with different field profiles. It is anyway **strong**, some 0,1% per K degree.  
For Si it is about  $\approx 30 \text{ mV/K}$  in devices with  $V_B = 30 \text{ V}$  and  $\approx 900 \text{ mV/K}$  in devices with  $V_B = 300 \text{ V}$ .

- In reality, the **breakdown current is not divergent** and flows without requiring a primary injected current. In fact the current is self-sustaining, because of the positive feedback intrinsic in the avalanche ionization process.
- What keeps finite the avalanche current is the **feedback effect due to the mobile space charge**. The effect is negligible for  $V_a < V_B$  (hence it is not taken into account in the former equations), but it is enhanced by the current rise at  $V_a > V_B$  and reduces the electric field that acts on the carriers. The multiplication thus stabilizes itself at the self-sustaining level.
- For  $V_a > V_B$  the avalanche current  $I_a$  increases linearly with  $V_a$ , so that an **avalanche resistance  $R_a$**  can be defined:  $R_a = \Delta V_a / \Delta i_a$ .
- In fact,  $\Delta V_a$  produces a proportional increase of the electric field, which increases the impact ionization probability, hence the avalanche current. In turn, the current rise produces an increase of the space charge, which counteracts the effect of  $\Delta V_a$ . The current thus rises until it brings back to self-sustaining condition the avalanche multiplication; that is, the current increase  $\Delta i_a$  is proportional to the voltage increase  $\Delta V_a$ .

A photodiode biased at  $V_a$  **below** the breakdown voltage  $V_B$  but **close to it** provides **linear amplification** of the current by exploiting the avalanche carrier multiplication.

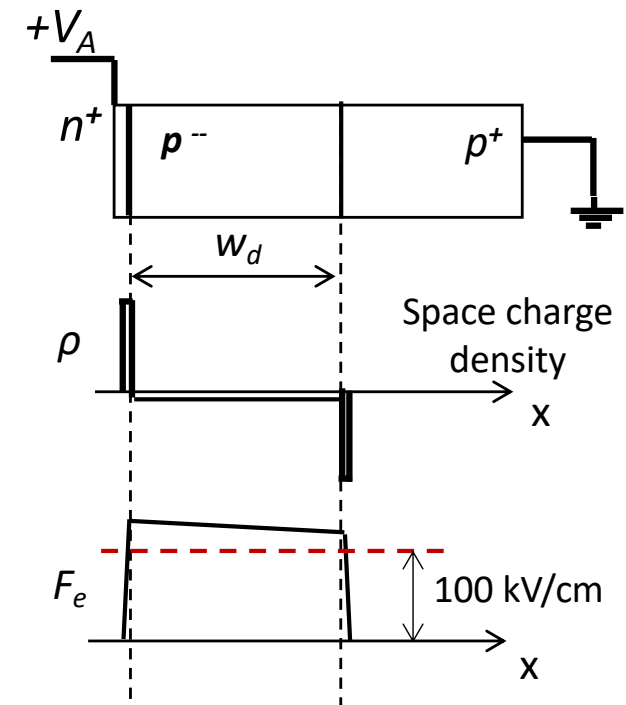
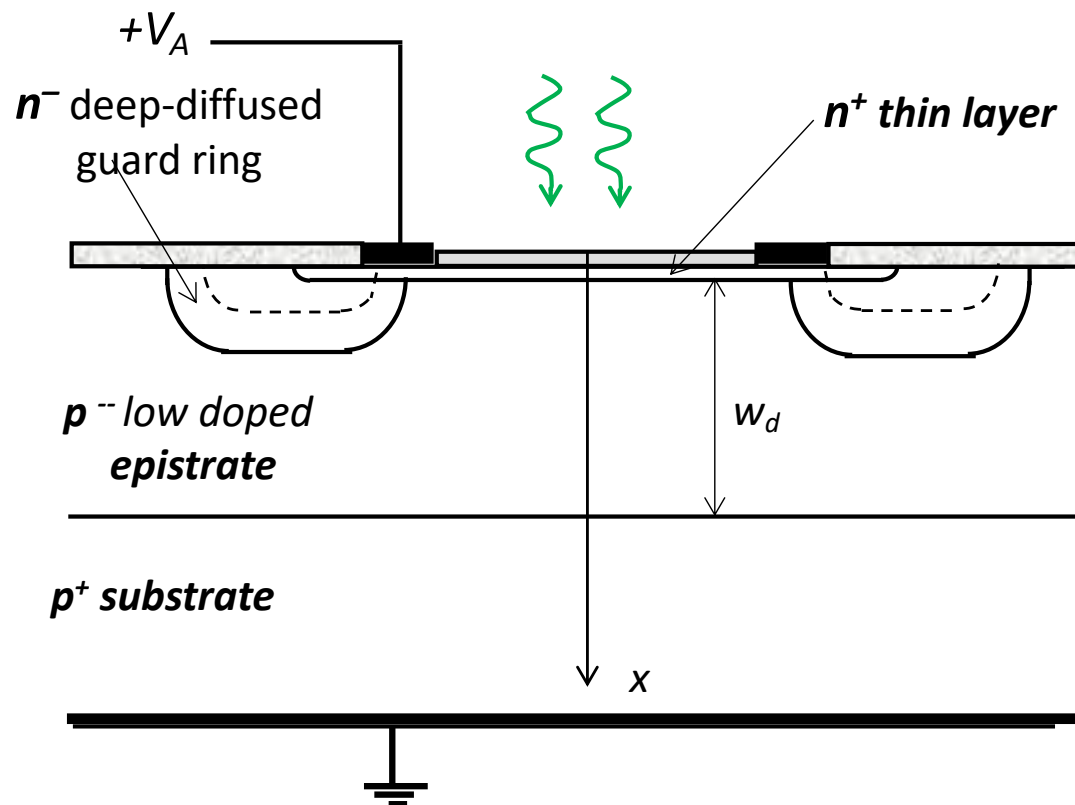
Such photodiodes with internal gain are called **Avalanche PhotoDiodes (APD)**; they bear some similarity to PhotoMultiplier Tubes (PMT), but have remarkably different features

- The amplification gain is the multiplication factor  $M$ , which can be adjusted by adjusting the bias voltage  $V_a$  with respect to  $V_B$
- Since  $V_B$  strongly depends on the diode temperature  $T$ , variations of  $T$  have effect equivalent to significant variations of the bias  $V_a$ . Therefore, for having a **stable gain  $M$** , **the temperature of the APD must be stabilized.**
- The actual dependance of  $M$  on  $V_a$  can be fitted fairly well by an **empirical** equation

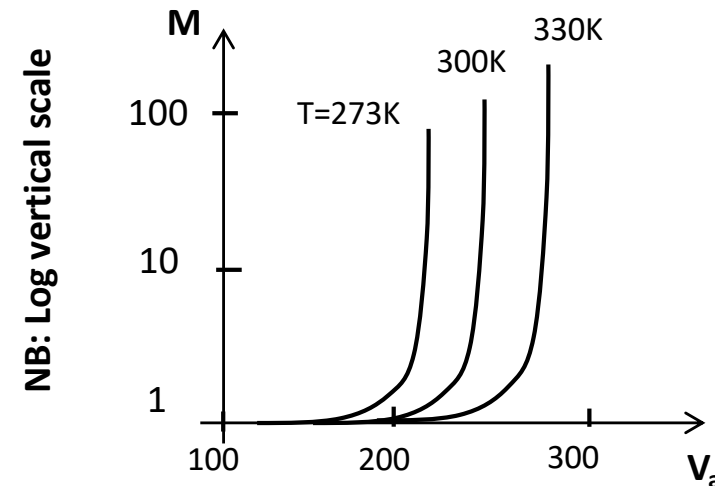
$$M = \frac{1}{1 - \left(\frac{V_a}{V_B}\right)^u}$$

- with exponent  $u$  that depends on the field profile (and on the type of semiconductor); it varies from 3 to 6, with higher values corresponding to wider high-field zone.

Early attempts to develop APDs exploited **PIN structures modified** for operating at higher electric field (typically  $F_e > 100 \text{ kV/cm}$ ): more efficient guard-ring for avoiding edge breakdown; higher uniformity in material processing over the sensitive area; etc. The PIN structure, however, turned out to be unsuitable for APD devices.



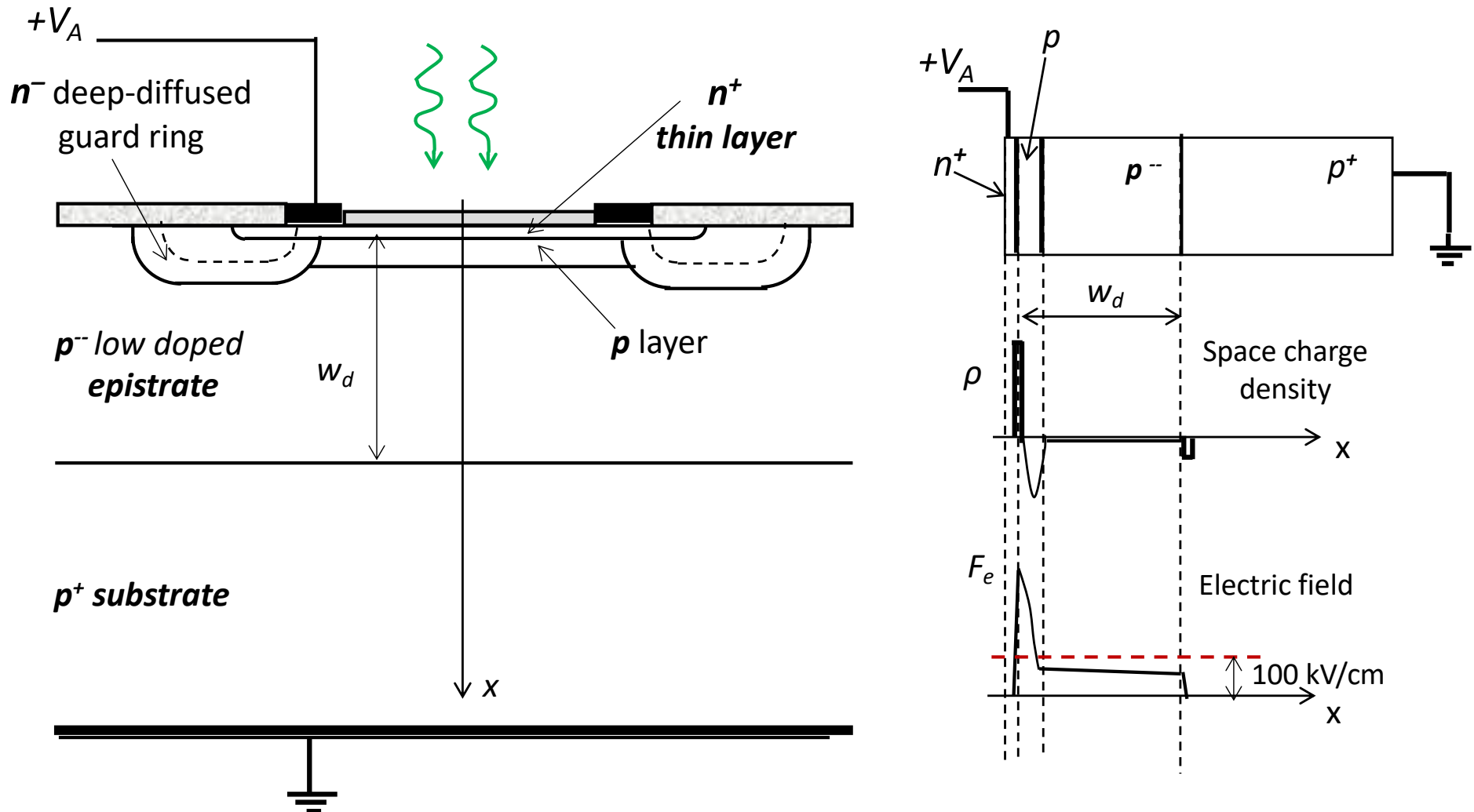
1. Even perfect p-i-n devices would have features not well suitable for operating as APD
2. Moreover, real p-i-n devices have unavoidable small local defects that rule out any prospect as APD.



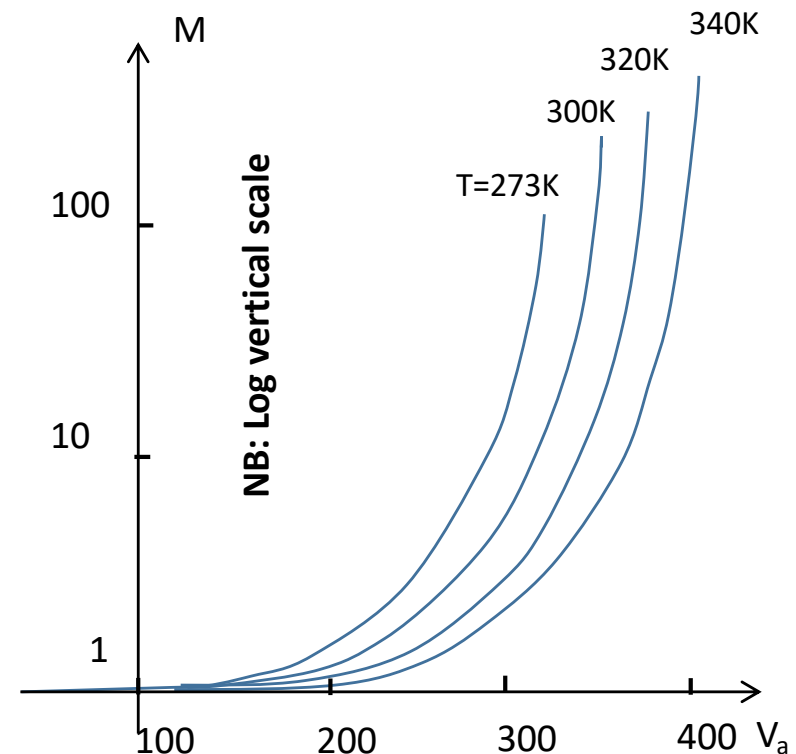
- Even a perfect p-i-n diode would have multiplication factor  $M$  very steeply rising with the bias voltage  $V_a$ , because the depletion layer is wide (for obtaining high detection efficiency) and the high electric field zone covers it almost completely. **It would be extremely difficult to obtain a stable and accurately controlled gain  $M$ .**

The evolution of the device design from PIN to Reach-Through APD structure was then driven by the insight gained in the PIN-APD failure.

Basic idea: to improve the structure by inserting a **thin layer with high electric field  $F_e$**  (where carriers undergo avalanche multiplication) beside a **wide depletion layer with moderate  $F_e$**  (where carriers just drift at saturated velocity)



- The total depletion layer width of Si RAPDs in most cases is from 10 to 30 $\mu\text{m}$ , in order to obtain high detection efficiency up to 800-900nm wavelength (NIR edge)
- The width of the multiplication region (where F exceeds the ionization threshold) is much thinner, from 1 to a few  $\mu\text{m}$
- Moderately steep rise of M with the bias voltage is obtained; the RAPD gain can thus be reliably controlled.
- The dependance of M on the device temperature is still remarkable and must be taken into account



**The highest M obtained with Si-APDs is much lower than the gain level currently provided by PMTs. In the best cases M values up to about 500 are obtained; attaining M=1000 is out of the question**



**Avalanche multiplication is a statistical process** → the APD gain has random fluctuations.

Let us denote by:

M the mean multiplication gain  
 $\sigma_M^2$  the gain variance and

In the multiplication, the fluctuations of the number of primary charges are not only amplified by  $M^2$ ; they are **further enhanced by a factor  $F > 1$**  called **Excess Noise Factor** (like for PMTs).

Input: primary carriers with  
mean number  $N_p$   
variance  $\sigma_p^2 = N_p$  (Poisson statistics)

Output: multiplied carriers with  
mean number  $N_u = M N_p$   
variance  $\sigma_u^2 = F M^2 \sigma_p^2 = F M^2 N_p$

The physical processes exploited for multiplying electrons in PMTs and in APDs are remarkably different and the detector gain has remarkably different features.

- In **PMTs**, the accelerated electron that hits a dynode is lost and the number of emitted secondary electrons fluctuates in a set of values that includes zero. The resulting mean number of carriers coming from the dynode is just the mean number of emitted secondary electrons and is definitely **higher than unity**.
- In **APDs**, the accelerated electron that undergoes a ionizing impact is not lost, it remains available for further impacts; the generation of a further electron (plus a hole) is statistical and the mean number of generated electrons is definitely **lower than unity**. The resulting mean number of electrons after the impact is **one plus the mean number of generated** electrons.
- In **PMTs** the gain is produced by an unidirectional sequence of events, the cascade of statistical multiplications at the various dynodes. Cascaded statistical processes can be well analyzed by known mathematical approaches (as the Laplace probability generating function)
- In **APDs** the **statistical process is much more complicated** than a simple cascade because of the intrinsic **positive feedback** in the impact-ionization. Rather than a cascade, it is a complex of interwoven feedback loops, each one originating from the other type of carrier (the hole in our case) generated in the impact.

**In Silicon with electric field intensity just above the ionization threshold**, the situation is very favorable since the  $F$  degradation due to the positive feedback is negligible.

- The ratio of ionization coefficients is very small  $k = \beta/\alpha < 0,01$   
→ probability of impact ionization by holes much lower than that of electrons.
- the mean number  $\mu$  of secondary electrons generated by the impact of an electron is **small  $\mu \ll 1$**

The process can be analyzed as a cascade of electron impacts. By employing the Laplace probability generating function and numbering in sequence the impacts we get

$$F = 1 + v_M^2 = 1 + \frac{1}{1 + \mu} \approx 2$$

**$F=2$  is the lowest possible  $F$  for Si-APDs and is achieved at low gain level. The conclusion is confirmed by experiments on carefully designed APD devices operating at  $M < 50$ .**

**For comparison, recall that ordinary PMTs routinely offer  $F < 2$  at very high gain  $M > 10^5$ .**

- Silicon with electric field just above the ionization threshold is a specially favorable case. In all other cases **the positive feedback in the avalanche process is remarkable**, it cannot be neglected and has detrimental effect on the variance of the APD gain.
- The fluctuation of the electrons generated in an impact is not only amplified by the further electron impacts in the subsequent multiplication path. The holes that are generated in the impact travel back and re-inject the fluctuation in a previous step of the multiplication path.
- This **back-injection of fluctuations enhances the excess noise factor  $F$** , with an efficiency that **increases with the  $k$  factor** (the relative ionization efficiency of holes versus electrons).
- **In Silicon the  $k$  factor markedly increases as the field is increased.** Therefore,  $F$  markedly increases as the bias voltage of the APD is raised for increasing the gain.

A thorough mathematical treatment of the avalanche multiplication is quite complicated and beyond the scope of this course. We will just comment some results of treatments reported in the technical literature.

With some simplifying assumptions (uniform electric field; constant  $k$  value), it has been shown that the excess noise factor  $F$  **with primary current of electrons** is

$$F \approx M \left[ 1 - (1 - k) \left( 1 - \frac{1}{M} \right)^2 \right]$$

- In cases with negligible positive feedback  $k=0$ , the equation confirms the result of the approximate analysis

$$F = 2 - \frac{1}{M} \approx 2 \quad (\text{since } M \gg 1)$$

- In cases with full positive feedback (i.e. equally efficient carriers, as in GaAs and other III-V semiconductors) it is  $k \approx 1$  and  $F$  increases as  $M$

$$F \approx M$$

- In cases with intermediate feedback level it is  $0 < k < 1$  and the equation specifies how  $F$  increases with  $M$  with rate of rise that increases with  $k$ . For instance:

with  $k=0,01$  at  $M=100$  we get  $F \approx 3$

with  $k=0,1$  at  $M=100$  we get  $F \approx 12$

- The gain  $M$  of the APD is intended to **bring signal and noise of the detector to a level higher than the noise of the following circuits**, with the aim of attaining better sensitivity (smaller optical signal) than a PIN photodiode (limited by the circuit noise)
- However, when the voltage is raised for increasing  $M$  **also the variance of the gain fluctuations increases**. At some level  $M_{max}$  the effect of the gain fluctuations becomes greater than that of the circuit noise: increasing  $M$  beyond this level would be nonsense. This  $M_{max}$  limit depends on the actual case (actual APD and circuit).
- It is the **maximum factor  $F_{max}$  tolerable in the actual case that actually determines the  $M_{max}$  level**. In critical cases (typically InGaAs APDs, which have  $F \approx M$ ) a fairly high value  $F_{max}$  turns out to be tolerable, even up to  $F_{max} \approx 10$ .

Thanks to the low  $k$  factor, Silicon devices have the lowest excess noise among APDs and achieve the highest gain levels.

Si-APD devices specially designed for low  $k$  have

$$F \leq 2,5 \quad \text{up to } M \approx 100$$

$$F \leq 5 \quad \text{up to } M \approx 500.$$

Ordinary Si-APD devices have fairly lower performance, i.e. typically

$$F \leq 4 \quad \text{up to } M \approx 100.$$

In III-V semiconductors (GaAs, InP, InAlAs, etc.) the ionization efficiencies of electrons and holes are equal ( $k=1$ ) or at least comparable ( $k\approx 1$ ). The positive feedback thus is very strong and  $F$  increases as  $M$  (see previous slides).

For InP-InGaAs and other III-V devices the useful gain range is fairly limited , typically:

$$F \leq 10 \text{ up to } M \approx 10$$

Nevertheless, InGaAs-APDs are in general preferred to Ge-APDs for detecting IR optical signals because they have lower dark-current (lower detector noise) and higher quantum detection efficiency, with cutoff to extended to longer wavelength (typically  $\lambda \leq 1,7 \mu\text{m}$ )