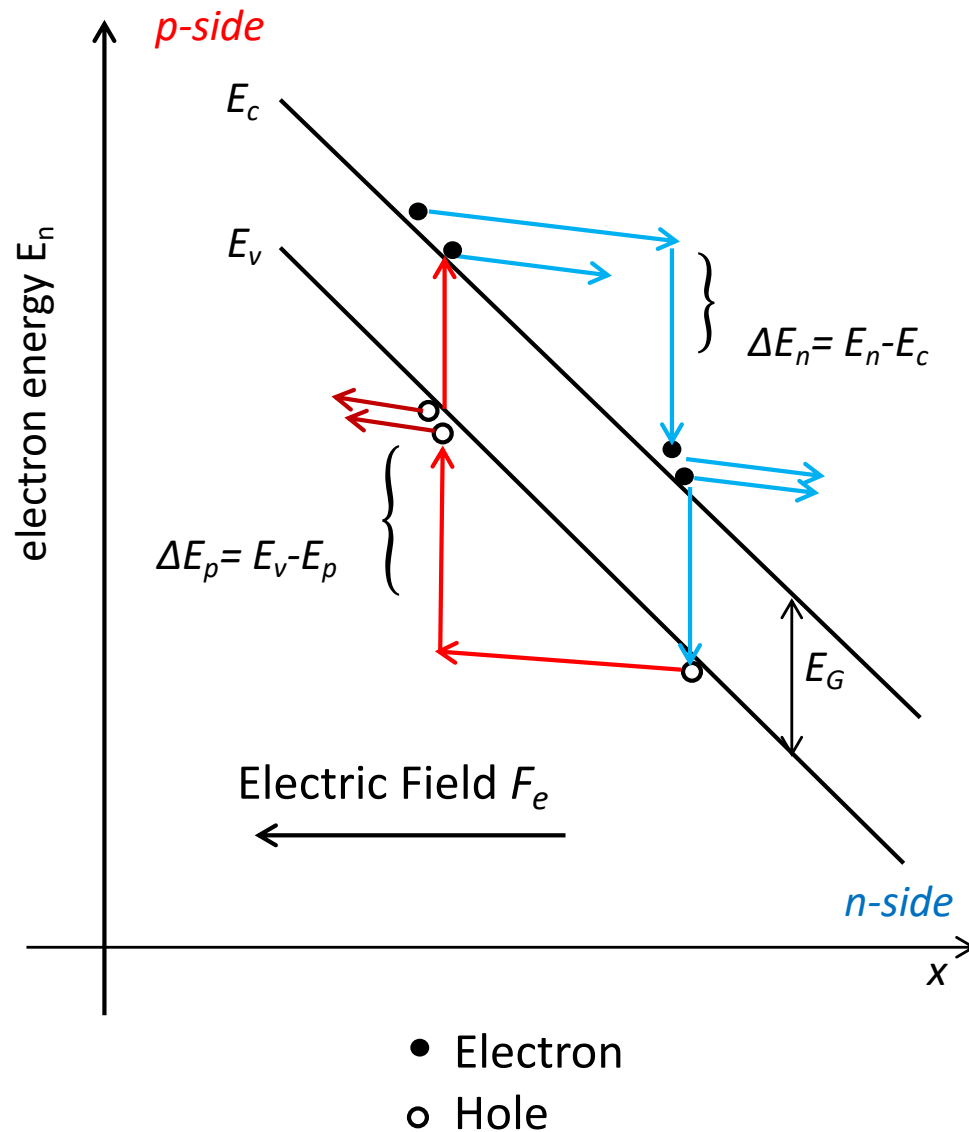


COURSE OUTLINE

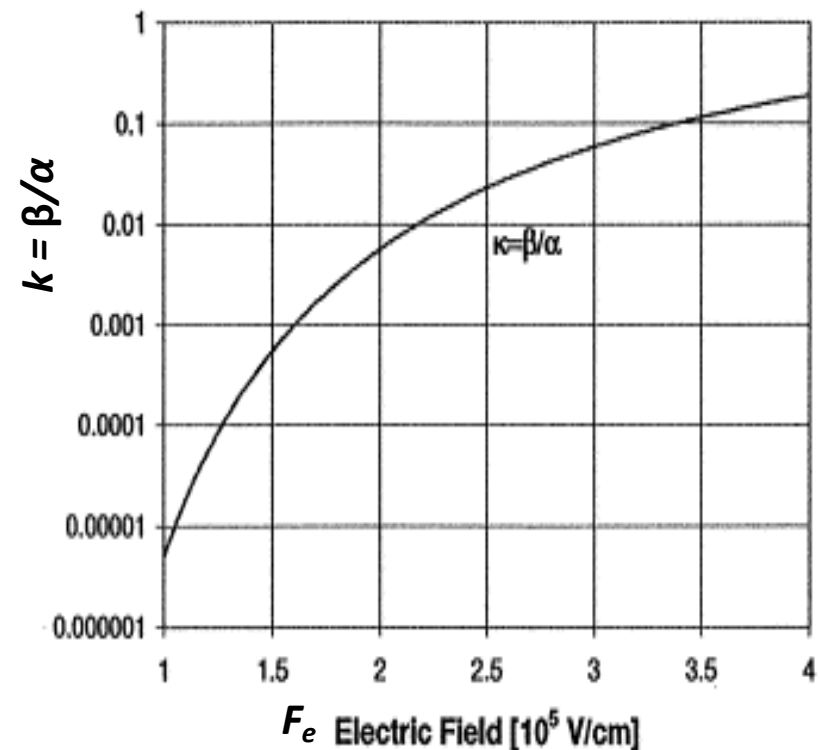
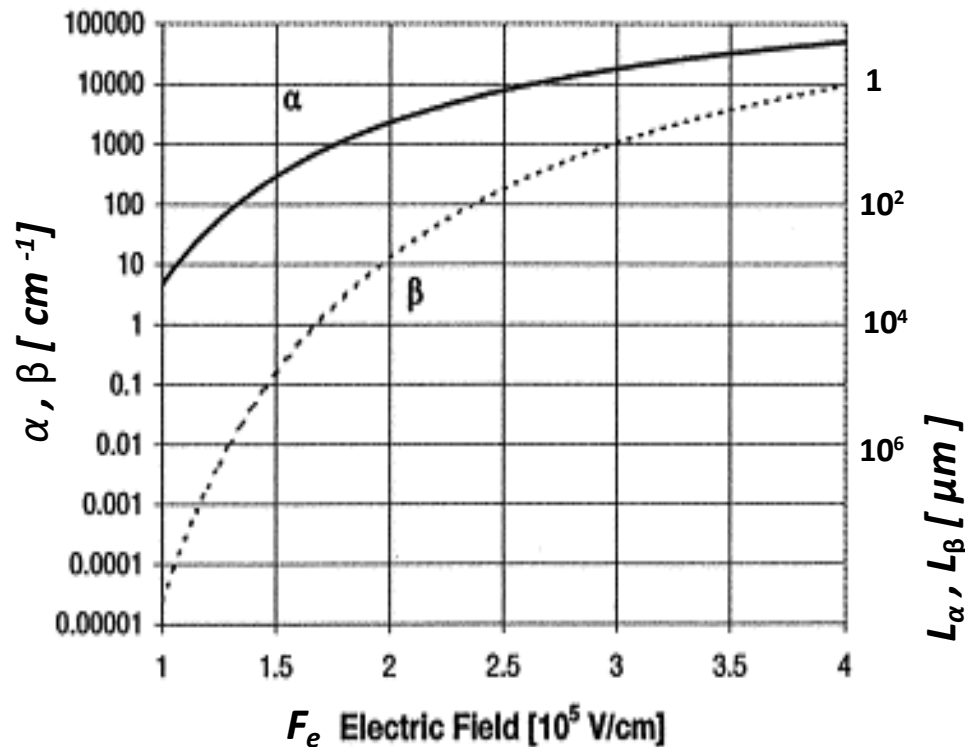
- Introduction
- Signals and Noise
- Filtering
- Sensors: PD5 – Avalanche PhotoDiodes

- Impact ionization in semiconductors
- Linear amplification by avalanche multiplication of free carriers
- Silicon Avalanche Photodiodes (Si-APD) and the device structure
- Statistical behavior of the avalanche multiplication and limits to the gain



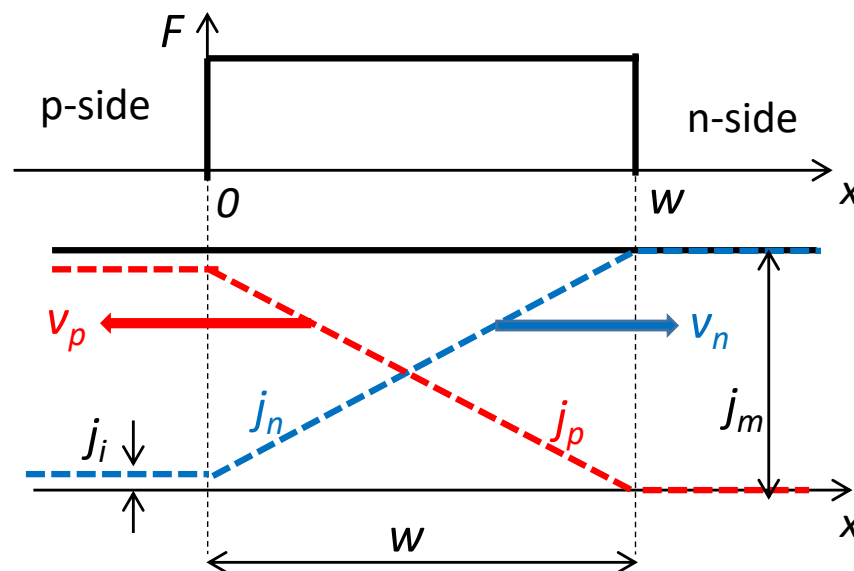
- ❑ A free electron drifting in the field gains kinetic energy $\Delta E_n = E_n - E_c$
- ❑ Part of ΔE_n is transferred to lattice vibrations by scattering events
- ❑ Because of energy and momentum conservation, a ionizing collision can occur only when
$$\Delta E_n > 1,5E_G$$
- ❑ Until reaching such ΔE_n the carrier travels without ionizing. The carrier multiplication thus has a dead-space; it is a **discontinuous statistical** process
- ❑ There is inherently a **positive feedback** loop in the process, because also holes can ionize by impact
- ❑ a cascade of ionizing collisions produces **avalanche multiplication** of carriers

- The carrier multiplication can be analyzed with a **continuous statistical model**, based on the **average in space of the true discontinuous random process**.
- The continuous model provides a good approximation if the width of the multiplication region (high-field region) is definitely larger than the mean path between ionizing collisions. **The model is inadequate if the high-field region is very thin**, i.e. for width smaller than or comparable to the mean path between collisions.
- The **ionizing coefficients α for electrons and β for holes** are defined as the **probability density of ionization in the carrier path**; that is, for a carrier traveling over dx the probability of producing impact-ionization in dx is
 αdx for electrons and βdx for holes
- The mean path between ionizing collisions thus is
 $L_\alpha = 1 / \alpha$ for electrons and $L_\beta = 1 / \beta$ for holes
- **The features of the multiplication process strongly depend on the relative intensity of the positive feedback, hence on the value of $k = \beta/\alpha$** , which is different in different materials:
- $k \ll 1$ in Silicon, $k > 1$ in Ge and $k \approx 1$ in GaAs and other III-V materials



- α and β rapidly increase with the electric field F_e . They can be described with good approximation by $\alpha = \alpha_o \exp\left(-\frac{F_{no}}{F_e}\right)$ and $\beta = \beta_o \exp\left(-\frac{F_{po}}{F_e}\right)$
In Silicon $\alpha_o = 3,8 \cdot 10^6 \text{ cm}^{-1}$, $F_{no} = 1,75 \cdot 10^6 \text{ V/cm}$; $\beta_o = 2,25 \cdot 10^7 \text{ cm}^{-1}$, $F_{po} = 3,26 \cdot 10^6 \text{ V/cm}$
- k is $\approx 0,1$ at high electric field F_e and as F_e decreases k strongly decreases (because the dynamics of valence-band holes and conduction-band electrons are different)
- α and β markedly decrease as temperature increases (because stronger lattice vibrations drain more energy from carriers in the path between ionizing collisions)

- Even employing the continuous model, **the complete mathematical analysis of the avalanche multiplication of carriers is quite complicated** and will not be reported.
- However, the basic features of avalanche diodes can be clarified by analyzing a **simple case**. In a **PIN junction with uniform and constant field** higher than the impact ionization threshold, let us consider the stationary avalanche current due to the injection from the p-side of a small primary current of electrons j_i



Note that:

1. e-h pairs are generated, hence there are **both** electron and hole currents, even in case the ionization by holes be negligible (i.e. $\beta \approx 0$)
2. The total current is constant $j_m = j_n + j_p$
3. The p and n carriers of the avalanche form a **dipole-like mobile space-charge** (mostly p at p-side, mostly n at n-side) that adds a **field opposite to the junction field** (due to the fixed ion space charge)

Carrier multiplication

In the simplest case $\alpha = \beta$ (e.g. in GaAs) the equation is simply and we obtain:

$$j_m = \frac{j_i}{1 - \int_0^w \alpha(x) dx} = \frac{j_i}{1 - I_i}$$

$$M = \frac{j_m}{j_i} = \frac{1}{1 - I_i}$$

$I_i = \int_0^w \alpha(x) dx$ is called **ionization integral** and has a clear physical meaning:

it is the probability for a carrier to have an ionizing collision in the path from $x=0$ to $x=w$

The current j_m is the primary current j_i amplified by the **multiplication factor M**

In cases with $\alpha \neq \beta$ the equation can still be written in the form

$$j_m = \frac{j_i}{1 - \int_0^w \alpha_e(x) dx} = \frac{j_i}{1 - I_i}$$

$$M = \frac{j_m}{j_i} = \frac{1}{1 - I_i}$$

but the ionization integral I_i is now the integral of an effective ionization coefficient α_e

$$\alpha_e = \alpha \exp \left[- \int_0^w (\alpha - \beta) d\xi \right]$$

$$j_m = \frac{j_i}{1 - I_i}$$

$$M = \frac{j_m}{j_i} = \frac{1}{1 - I_i}$$

- The ionization integral I_i in any case **strongly depends** on the **applied bias voltage V_a** and on the **temperature T**
- I_i is nil until the field F_e produced by V_a attains level sufficient for impact ionization
- When the applied bias voltage V_a reaches a characteristic value V_B , the Ionization Integral $I_i \rightarrow 1$ and, according to the equation, $M \rightarrow \infty$ and $j_m \rightarrow \infty$
- V_B is called **Breakdown Voltage**; it is a characteristic feature of the diode, ruled by the distribution of the electric field F_e and by the dependance of α and β on the electric field F_e and on the temperature T
- V_B **increases with the temperature T** . The increase is different in devices with different field profiles. It is anyway strong, some 0,1% per K degree.

For Si it is about $\approx 30 \text{ mV/K}$ in devices with $V_B = 30 \text{ V}$ and $\approx 900 \text{ mV/K}$ in devices with $V_B = 300 \text{ V}$.

A photodiode biased at V_a **below the breakdown voltage V_B but close to it** provides **linear amplification** of the current by exploiting the avalanche carrier multiplication.

Such photodiodes with internal gain are called **Avalanche PhotoDiodes (APD)**; they bear some similarity to PhotoMultiplier Tubes (PMT), but have remarkably different features

- **The amplification gain is the multiplication factor M** , which can be adjusted by adjusting the bias voltage V_a with respect to V_B
- Since V_B strongly depends on the diode temperature T , variations of T have effect equivalent to significant variations of the bias V_a . Therefore, for having a **stable gain M** , **the temperature of the APD must be stabilized**.
- The actual dependance of M on V_a can be fitted fairly well by an **empirical** equation

$$M = \frac{1}{1 - \left(\frac{V_a}{V_B}\right)^u}$$

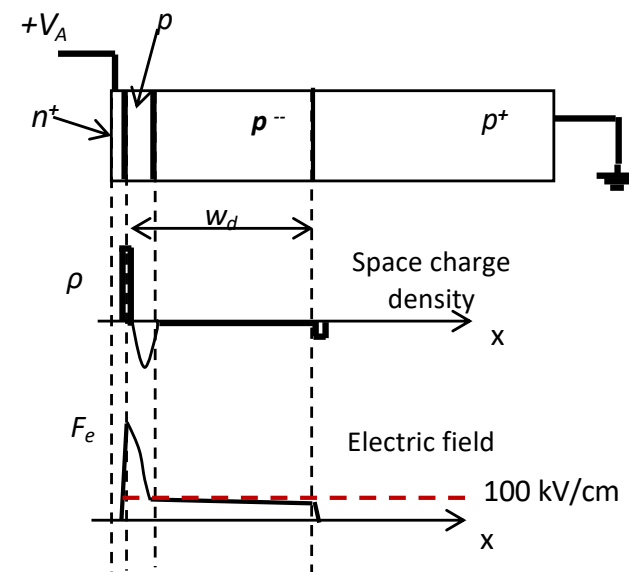
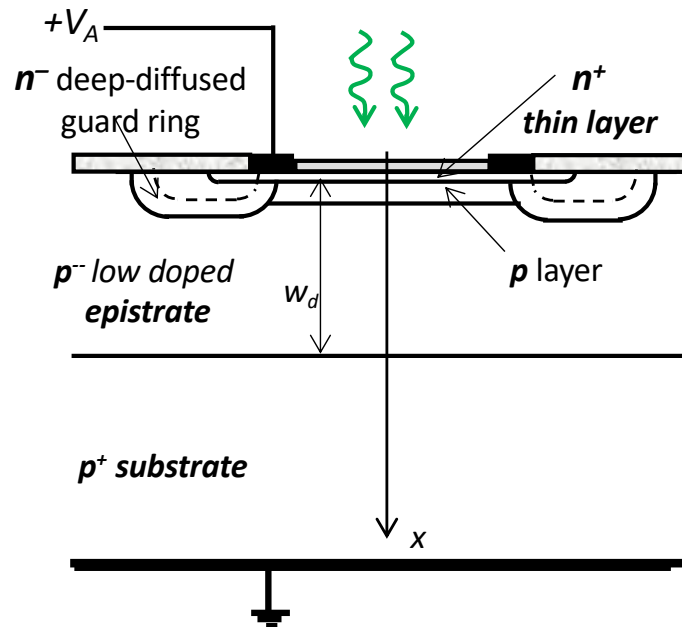
- with exponent u that depends on the field profile (and on the type of semiconductor); it varies from 3 to 6, with higher values corresponding to wider high-field zone.

The PIN structure, however, turned out to be unsuitable for APD devices.

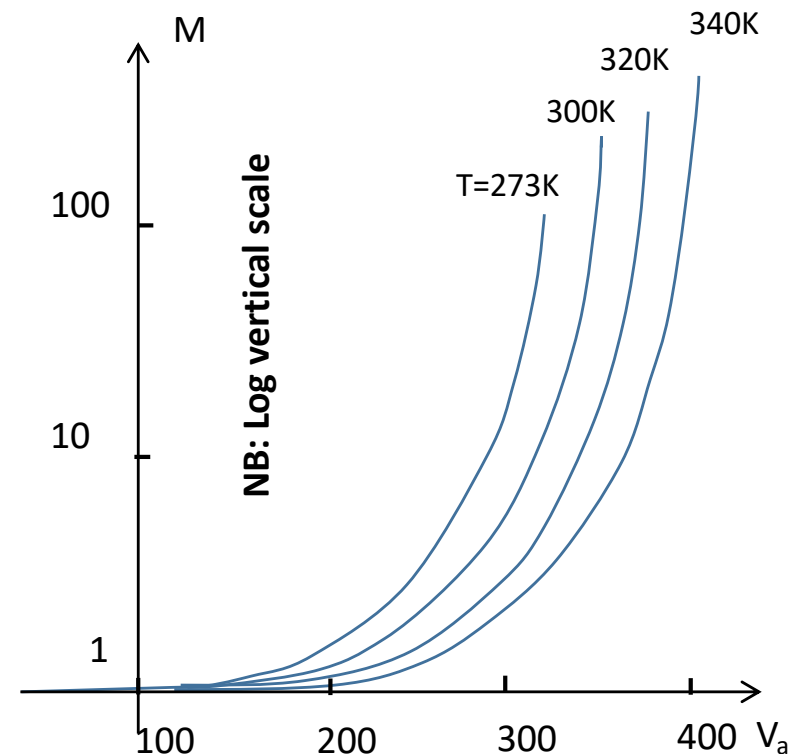
Even a perfect p-i-n diode would have multiplication factor M very steeply rising with the bias voltage V_a , because the high electric field zone covers it almost completely. **It would be extremely difficult to obtain a stable and accurately controlled gain M .**

The evolution of the device design from PIN to Reach-Through APD structure was then driven by the insight gained in the PIN-APD failure.

Basic idea: to improve the structure by inserting a **thin layer with high electric field F_e** (where carriers undergo avalanche multiplication) beside a **wide depletion layer with moderate F_e** (where carriers just drift at saturated velocity)



- The total depletion layer width of Si RAPDs in most cases is from 10 to 30 μm , in order to obtain high detection efficiency up to 800-900nm wavelength (NIR edge)
- The width of the multiplication region (where F exceeds the ionization threshold) is much thinner, from 1 to a few μm
- **Moderately steep rise of M with the bias voltage** is obtained; the RAPD gain can thus be reliably controlled.
- **The dependance of M on the device temperature is still remarkable** and must be taken into account



The highest M obtained with Si-APDs is much lower than the gain level currently provided by PMTs. In the best cases M values up to about 500 are obtained; attaining $M=1000$ is out of the question

Avalanche multiplication is a statistical process → the APD gain has random fluctuations.

Let us denote by:

M the mean multiplication gain

σ_M^2 the gain variance and

In the multiplication, the fluctuations of the number of primary charges are not only amplified by M^2 ; they are **further enhanced by a factor $F > 1$** called **Excess Noise Factor** (like for PMTs).

Input: primary carriers with

Mean number N_p

Variance $\sigma_p^2 = N_p$ (Poisson statistics)

Output: multiplied carriers with

Mean number $N_u = M N_p$

Variance $\sigma_u^2 = F M^2 \sigma_p^2 = F M^2 N_p$

The physical processes exploited for multiplying electrons in PMTs and in APDs are remarkably different and the detector gain has remarkably different features.

- In **PMTs**, the accelerated electron that hits a dynode is lost and the number of emitted secondary electrons fluctuates in a set of values that includes zero. The resulting mean number of carriers coming from the dynode is just the mean number of emitted secondary electrons and is definitely **higher than unity**.
- In **APDs**, the accelerated electron that undergoes a ionizing impact is not lost, it remains available for further impacts; the generation of a further electron (plus a hole) is statistical and the mean number of generated electrons is definitely **lower than unity**. The resulting mean number of electrons after the impact is **one plus the mean number of generated** electrons.
- In **PMTs** the gain is produced by an **unidirectional sequence of events**, the cascade of statistical multiplications at the various dynodes. Cascaded statistical processes can be well analyzed by known mathematical approaches
- In **APDs** the **statistical process is much more complicated** than a simple cascade because of the intrinsic **positive feedback** in the impact-ionization. Rather than a cascade, it is a complex of interwoven feedback loops, each one originating from the other type of carrier generated in the impact.

In Silicon with electric field intensity just above the ionization threshold, the situation is very favorable since the F degradation due to the positive feedback is negligible.

- The ratio of ionization coefficients is very small **$k = \beta/\alpha < 0,01$**
→ probability of impact ionization by holes much lower than that of electrons.
- the mean number μ of secondary electrons generated by the impact of an electron is small **$\mu \ll 1$**

The process can be analyzed as a cascade of electron impacts. By employing the Laplace probability generating function and numbering in sequence the impacts we get

$$F = 1 + v_M^2 = 1 + \frac{1}{1 + \mu} \approx 2$$

$F=2$ is the lowest possible F for Si-APDs and is achieved at low gain level. The conclusion is confirmed by experiments on carefully designed APD devices operating at $M < 50$.

For comparison, recall that ordinary PMTs routinely offer $F < 2$ at very high gain $M > 10^5$.

- Silicon with electric field just above the ionization threshold is a specially favorable case. In all other cases **the positive feedback in the avalanche process is remarkable**, it cannot be neglected and has detrimental effect on the variance of the APD gain.
- This **back-injection of fluctuations enhances the excess noise factor F** , with an efficiency that **increases with the k factor**
- **In Silicon the k factor markedly increases as the field is increased**. Therefore, F markedly increases as the bias voltage of the APD is raised for increasing the gain.

A thorough mathematical treatment of the avalanche multiplication is quite complicated and beyond the scope of this course. We will just comment some results of treatments reported in the technical literature.

With some simplifying assumptions (uniform electric field; constant k value), it has been shown that the excess noise factor **F with primary current of electrons is**

$$F \approx M \left[1 - (1 - k) \left(1 - \frac{1}{M} \right)^2 \right]$$

- **In cases with negligible positive feedback $k=0$** , the equation confirms the result of the approximate analysis

$$F = 2 - \frac{1}{M} \approx 2 \quad (\text{since } M \gg 1)$$

- **In cases with full positive feedback** (as in GaAs and other III-V semiconductors) it is **$k \approx 1$** and F increases as M

$$F \approx M$$

- **In cases with intermediate feedback level it is $0 < k < 1$** and the equation specifies how F increases with M with rate of rise that increases with k . For instance:

with $k=0,01$ at $M=100$ we get $F \approx 3$

with $k=0,1$ at $M=100$ we get $F \approx 12$

- The gain M of the APD is intended to **bring signal and noise of the detector to a level higher than the noise of the following circuits**, with the aim of attaining better sensitivity than a PIN photodiode (limited by the circuit noise)
- However, when the voltage is raised for increasing M **also the variance of the gain fluctuations increases**. At some level M_{max} the effect of the gain fluctuations becomes greater than that of the circuit noise: increasing M beyond this level would be nonsense. This M_{max} limit depends on the actual APD and circuit.
- It is the **maximum factor F_{max} tolerable in the actual case that actually determines the M_{max} level**.

Thanks to the low k factor, Silicon devices have the lowest excess noise among APDs and achieve the highest gain levels.

Si-APD devices specially designed for low k have

$$F \leq 2,5 \quad \text{up to } M \approx 100$$

$$F \leq 5 \quad \text{up to } M \approx 500.$$

Ordinary Si-APD devices have fairly lower performance, i.e. typically

$$F \leq 4 \quad \text{up to } M \approx 100.$$

In III-V semiconductors (GaAs, InP, InAlAs, etc.) the ionization efficiencies of electrons and holes are equal ($k=1$) or at least comparable ($k\approx 1$). **The positive feedback thus is very strong** and F increases as M (see previous slides).

For InP-InGaAs and other III-V devices the useful gain range is fairly limited , typically:

$$F \leq 10 \text{ up to } M \approx 10$$

Nevertheless, InGaAs-APDs are in general preferred to Ge-APDs for detecting IR optical signals because they have lower dark-current and higher quantum detection efficiency, with cutoff to extended to longer wavelength (typically $\lambda \leq 1,7 \mu\text{m}$)