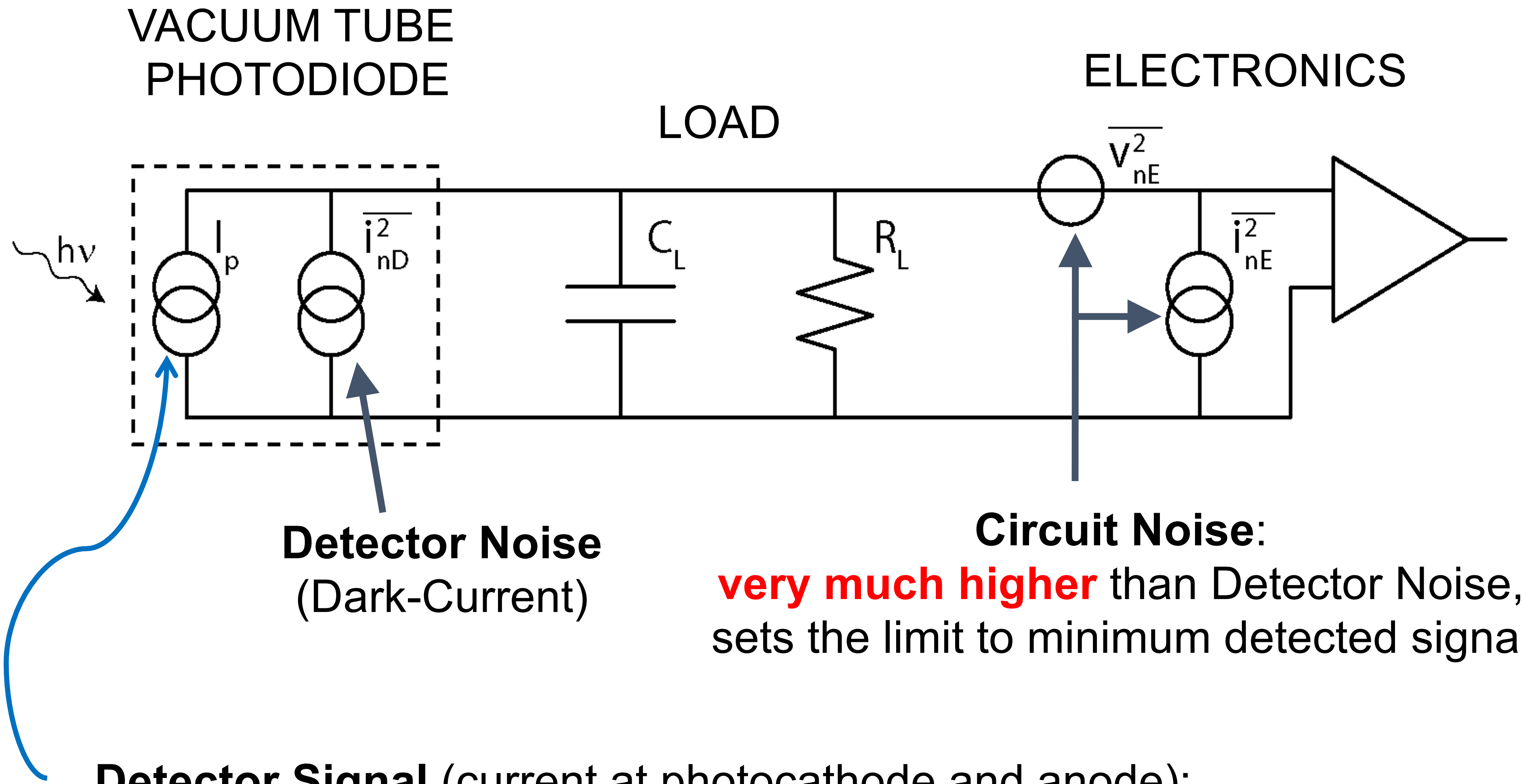
COURSE OUTLINE

- Introduction
- Signals and Noise
- Filtering
- Sensors: PD4 - PhotoMultiplier Tubes PMT

Photo Multiplier Tubes (PMT)

- Photodetectors that overcome the circuit noise
- Secondary Electron Emission in Vacuum and Current Amplification by a Dynode Chain
- Photo Multiplier Tubes (PMT): basic device structure and current gain
- Statistical nature of the current multiplier and related effects
- Dynamic response of PMTs
- Signal-to-Noise Ratio and Minimum Measurable Signal

Circuit Noise limits the sensitivity of photodiodes ...

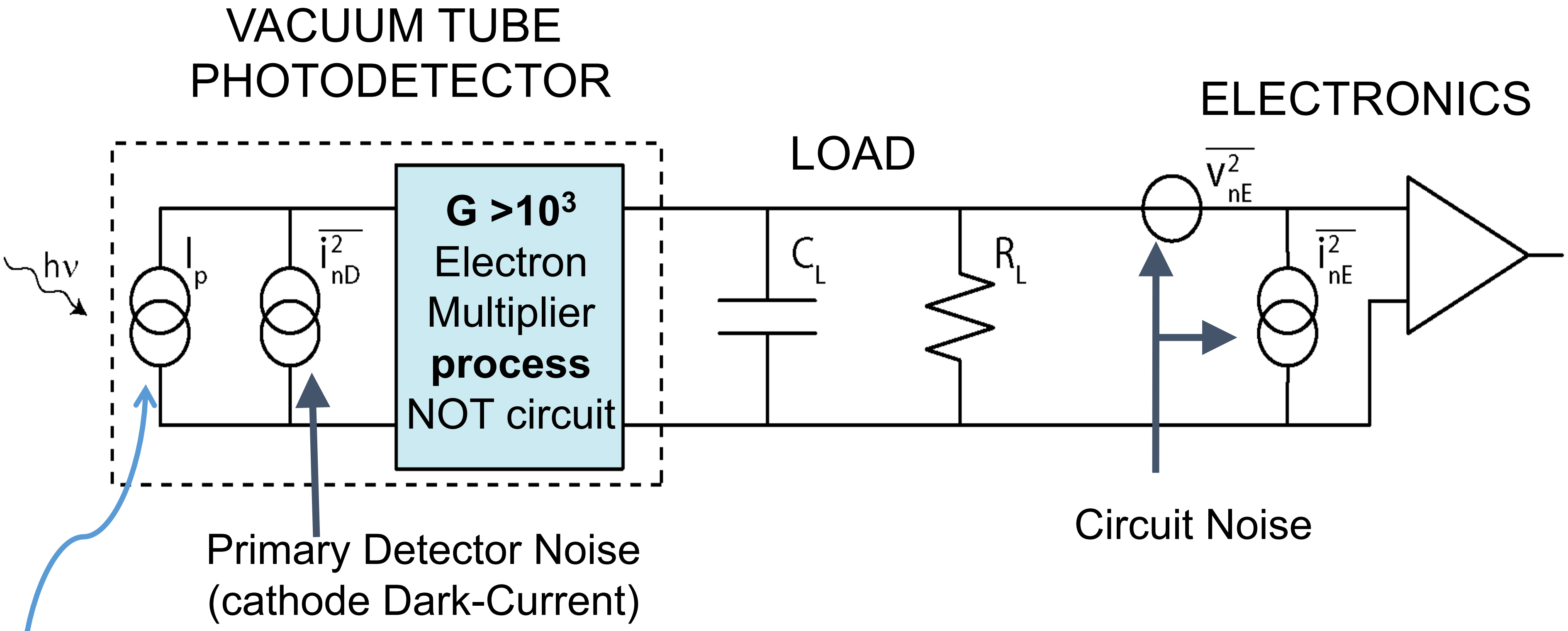


Detector Noise
(Dark-Current)

Circuit Noise:
very much higher than Detector Noise,
sets the limit to minimum detected signal

Detector Signal (current at photocathode and anode):
just one electron per detected photon!

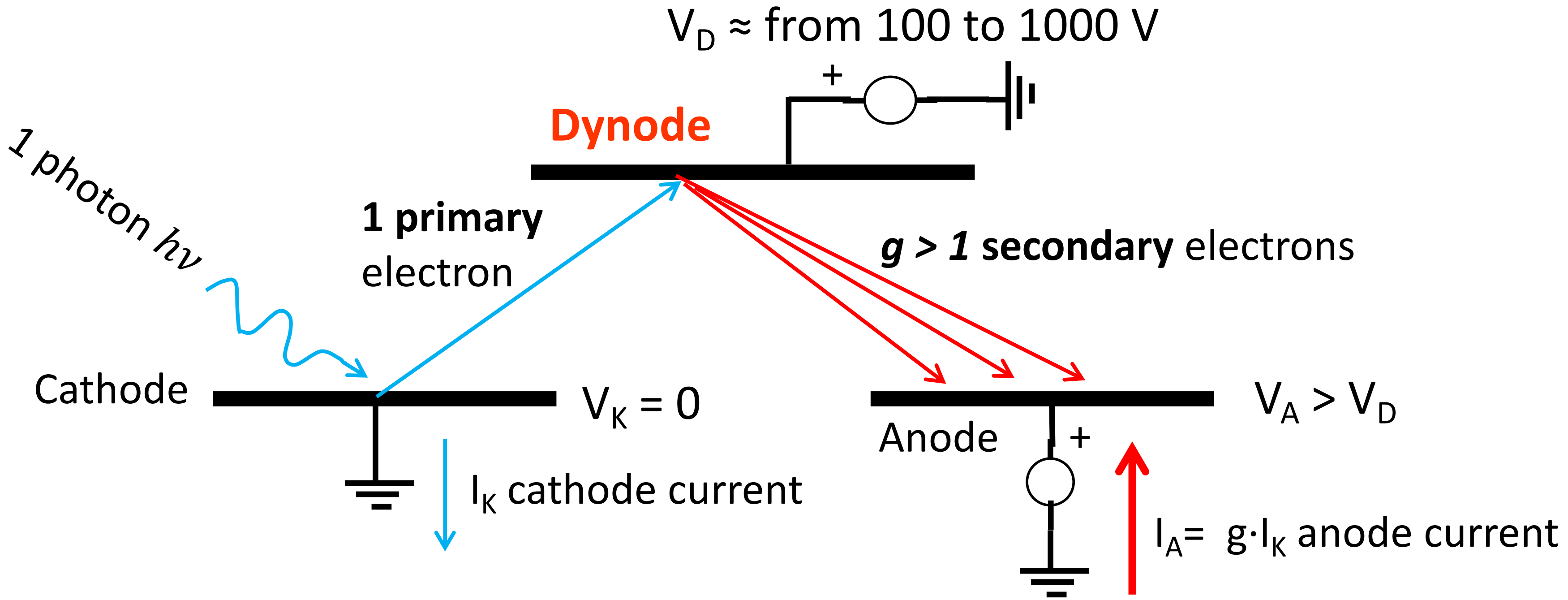
… but an Electron Multiplier Overcomes the Circuit Noise



- Primary Signal (photocathode current): one electron per detected photon
- Output (anode) current: **G >10^3** electrons per primary electron
- Dark-current noise and/or photocurrent noise at detector output are much higher than circuit noise, which has practically negligible effect

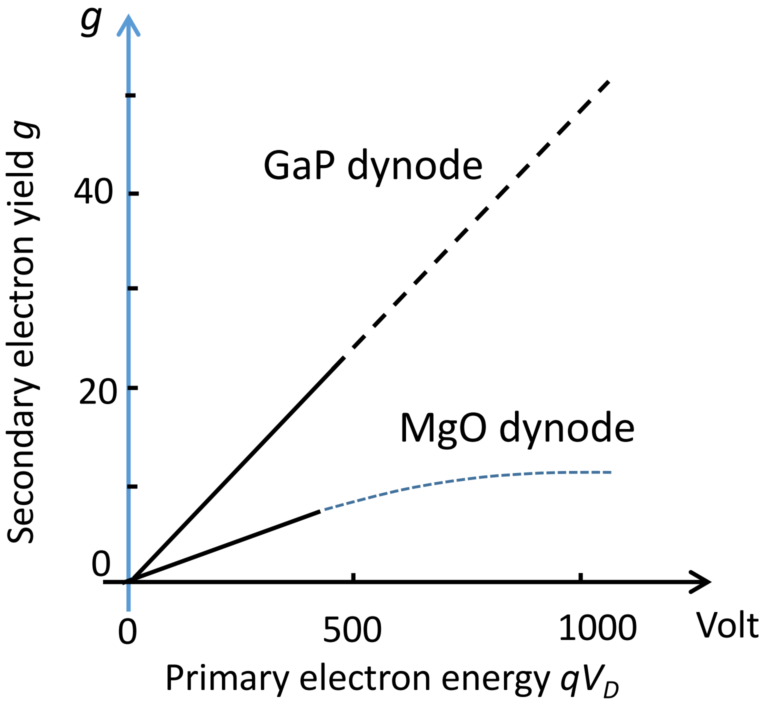
Secondary Electron Emission in Vacuum and Current Amplification by a Dynode Chain

Secondary Electron Emission in Vacuum



- A primary electron is emitted in vacuum with very little kinetic energy
- Driven in vacuum by a high potential difference (some 100V), it impacts with high energy on a **dynode** (electrode coated with suitable material, see later)
- Energy is transferred to electrons in the dynode; some of them gain sufficient energy to be emitted in vacuum; ***g > 1*** is the **yield** of secondary electrons per primary electron

Dynode materials



Secondary emitter coatings with ordinary yield:

- MgO Magnesium Oxide
- Cs_3Sb Cesium Antimonide
- BeO Beryllium Oxide
- Cu-Be Copper-Beryllium alloys

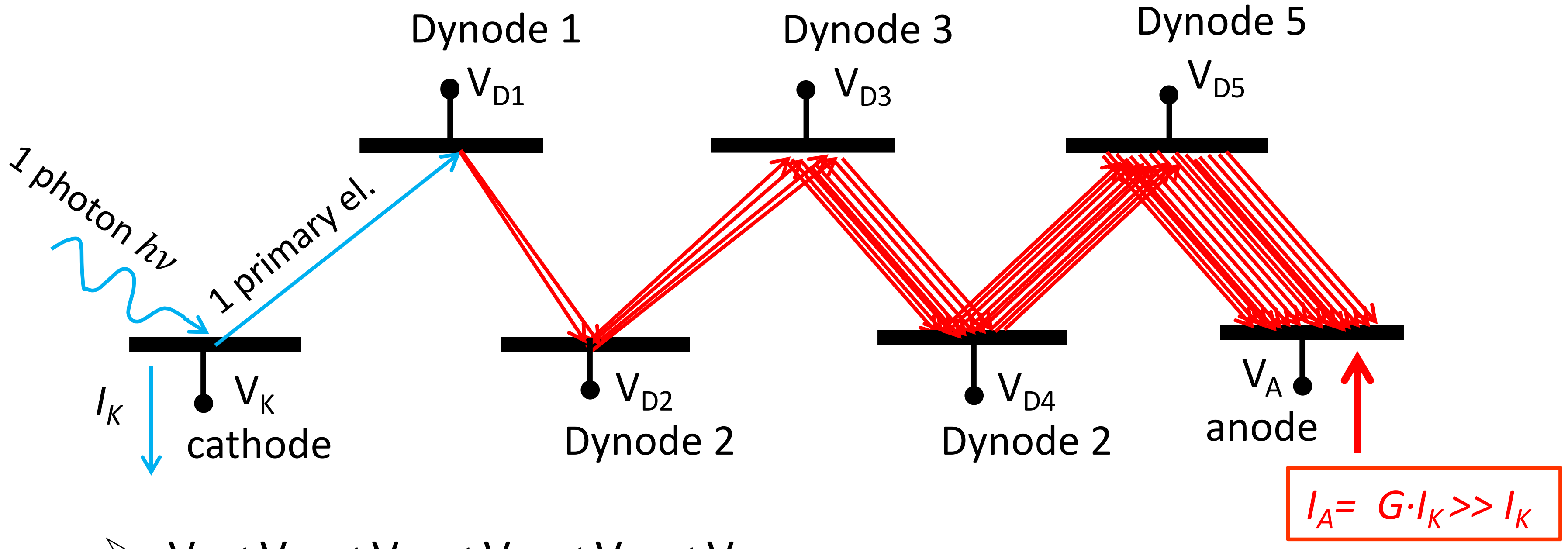
Secondary emitter with high yield:

- GaP Gallium Phosphide

- In the normal working range up to ≈500V, the emission yield *g* is **proportional** to the accelerating voltage V (i.e. the primary electron energy) $g = k_s V_D$
- At higher voltage *g* rises slower and tends to saturate (energy is transferred also to electrons in deeper layers, which have lower probability of escape in vacuum)
- In the linear range ordinary emitters work with ***g* values from ≈1,5 to ≈7** and GaP dynodes *g* values from ≈5 to ≈25
- **GaP dynodes are more costly and delicate**, require special care in operation and their yield tends to decrease progressively over long operation times

Electron Multiplier

Sketch of the Principle (example with 5 dynodes)



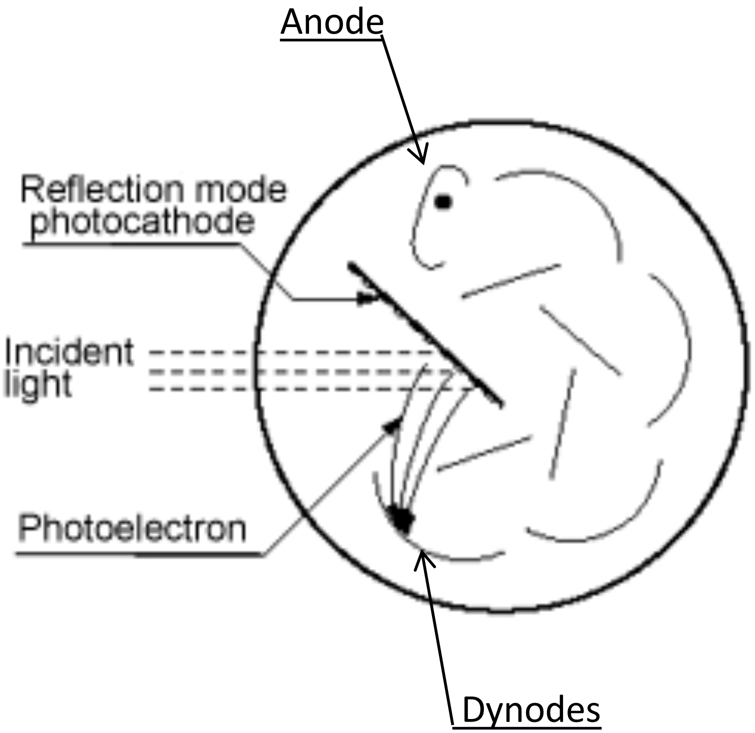
- $V_K < V_{D1} < V_{D2} < V_{D3} < V_{D5} < V_A$
- Electron optics (i.e. potential distribution) **carefully designed to lead the electrons** emitted from each electrode to the next one
- $g_r > 1$ secondary electron yield of dynode r
- $G = g_1 \cdot g_2 \cdot g_3 \cdot g_4 \cdot g_5$ **overall multiplier gain**

  that is, $G=g^5$ with equal stages $g_1=g_2=....=g$

Photo Multiplier Tubes (PMT): device structure and current gain

Side-on PhotoMultiplier Tubes PMTs

PMTs with side-window and opaque photocathode

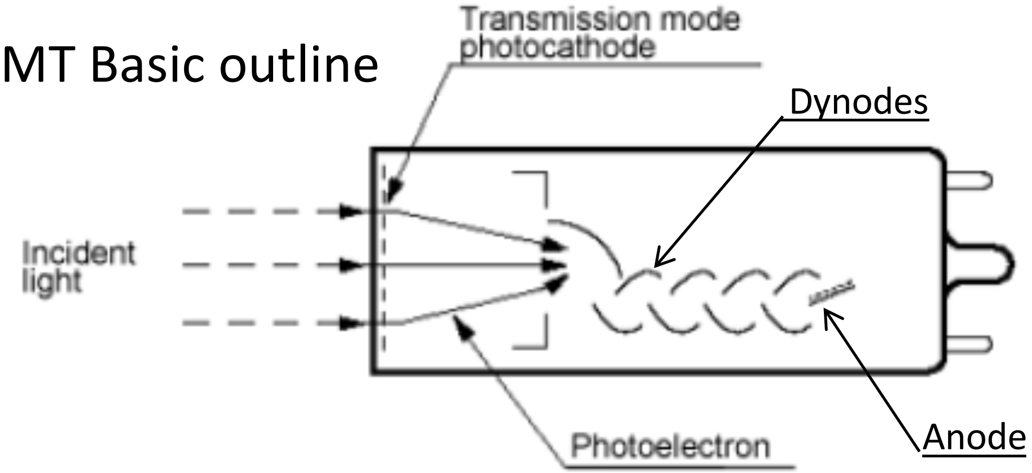


The basic structure of Photomultiplier Tubes with discrete dynodes and electrostatic-focusing was first demonstrated in 1937 by the RCA Laboratories; in the following decades it was progressively improved and developed by various industrial laboratories (RCA, DuMont, EMI, Philips, Hamamatsu...)

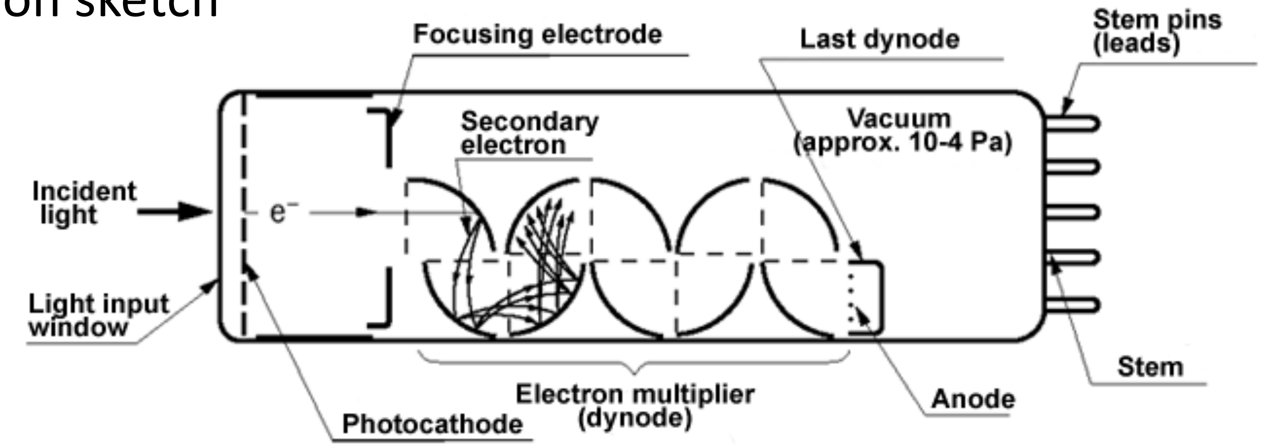
End-on PhotoMultiplier Tubes PMT

PMTs with end-window and semitransparent photocathode

PMT Basic outline



PMT Operation sketch



PMTs with semitransparent photocathode

Photocathode

Multiplier
dynode set

For comparison:
APD Avalanche Photodiode
mounted in case

APD Avalanche Photodiode
in silicon chip

PMT Gain Regulation and Stabilization

- PMTs can have high number *n* of dynodes (from 8 to 12) and attain high gain G.

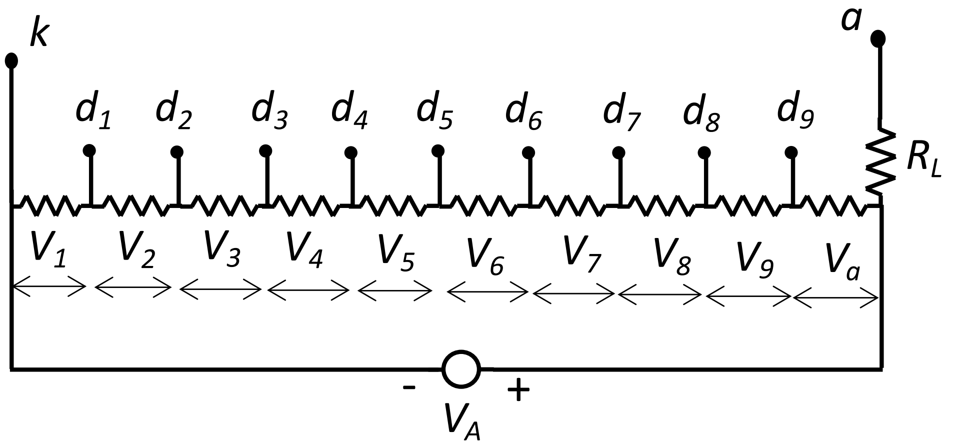
  With n equal dynodes it is $G = g^n$; e.g. with 12 dynodes $G=g^{12}$

  $G = 10^4$ with g = 2,2

  $G = 10^5$ with g = 2,6

  $G = 10^6$ with g = 3,2

- G is controlled by the dynode bias voltage, which regulates the dynode yield g
- A single supply is usually employed, with high voltage V_A typically from 1500 to 3000 V. The dynode voltages are obtained with a voltage-divider resistor chain; the potential difference V_j between two dynodes *j* and *(j-1)* is a preset fraction f_j of the supply V_A



$$\boxed{V_j = f_j\ V_A}$$

PMT Gain Regulation and Stabilization

- The supply voltage V_A thus rules the yield g_j of every dynode $g_j = k_S V_j = k_S f_j V_A$

  and the total gain $G = g_1 g_2 .... g_n = k_S V_1 \cdot k_S V_2 ..... k_S V_n = k_S^n f_1 f_2 ... f_n \cdot V_A^n$

  which increases with V_A **much more** than linearly

$$\boxed{G = k_S^n f_1 f_2 ... f_n \cdot V_A^n = K_G \cdot V_A^n}$$

  (NB: $K_G = k_S^n \, f_1 f_2 ... f_n$ is constant, set by the voltage distribution and dynode characteristics)

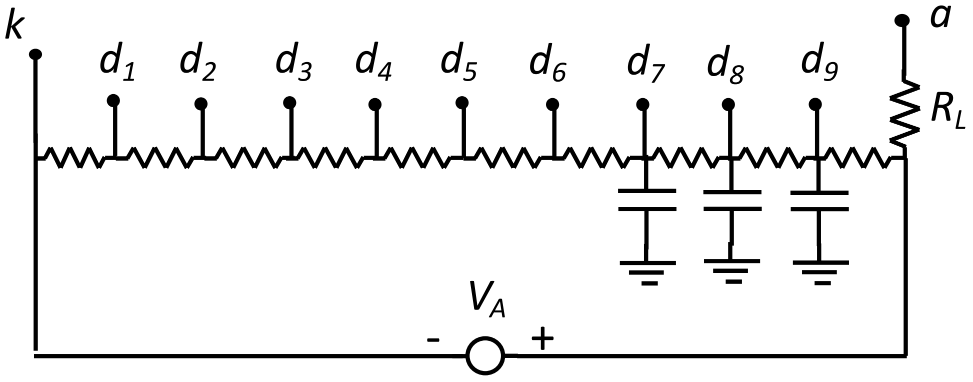
- The gain G is very sensitive to even small variations of the supply V_A : the relative variations of supply voltage are n-fold amplified in the relative variations of gain

$$\boxed{\frac{dG}{G} = n \frac{dV_A}{V_A}}$$

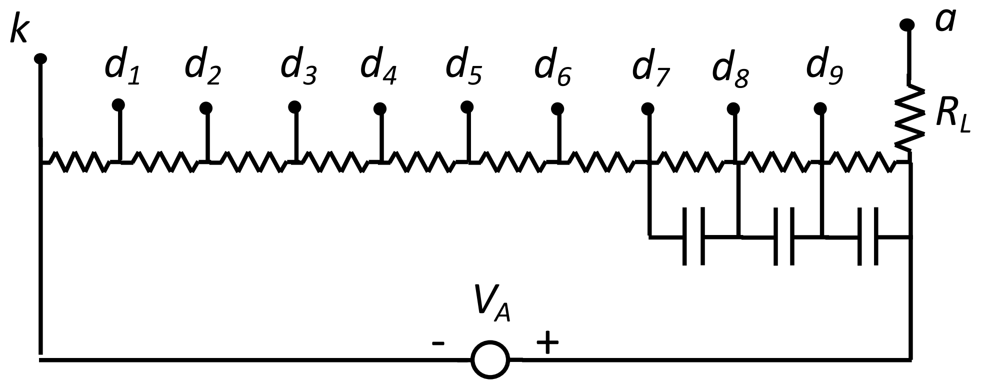
- Consequently, tight requirements must be set to the stability of the high voltage V_A versus ambient temperature and/or power-line voltage variations.
  e.g. getting ***G* stability better than 1%** for a PMT **with n=12 dynodes**
  requires a high voltage supply ***V_A* better stable than 0,08 %**

Cautions and limits in PMT exploitation

- For **limiting self-heating of voltage divider** below a few Watt, the divider current must be < 1 mA, hence total divider resistance must be at least a few MΩ.
- In order to avoid nonlinearity in the current amplification, variations of dynode voltages caused by the PMT current should be negligible. The PMT output current must thus be less than 1% of the divider current, i.e. typically a few μA.
- This limit is acceptable for DC current, but not for pulsed optical signals. However, fast transients of dynode voltages can be limited by introducing in the last stages capacitors in low-pass filtering configurations, as sketched in the examples



Cautions and limits in PMT exploitation

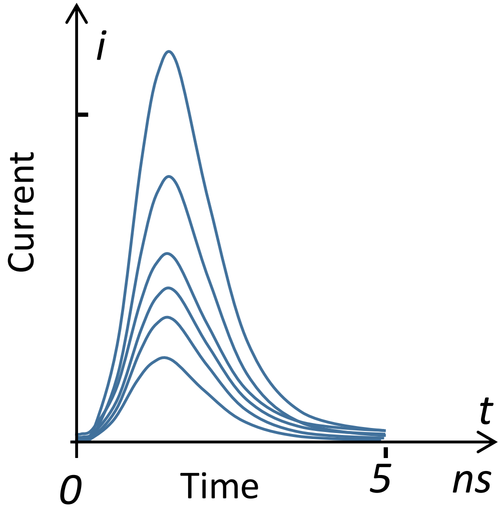


- **Space-charge effects may cause nonlinearities** in the amplification of fast pulsed signals. A high charge of the signal itself can significantly reduce the electric field that drives the electrons: the higher is the pulse, the slower gets the electron collection.
- Nonlinearity can occur also if the voltage signal developed on the load is high enough to reduce the driving field from last dynode to anode
- **Magnetic fields have very detrimental effect**: the electrons traveling in vacuum are deviated and the operation is inhibited or badly degraded. With moderate field intensity, magnetic screens can limit the effects; with high intensity fields PMT operation is actually impossible
- **PMTs are fairly delicate** and subject to fatigue effects and their operation is prejudiced by mechanical vibrations

Statistical nature of the electron multiplier and related effects

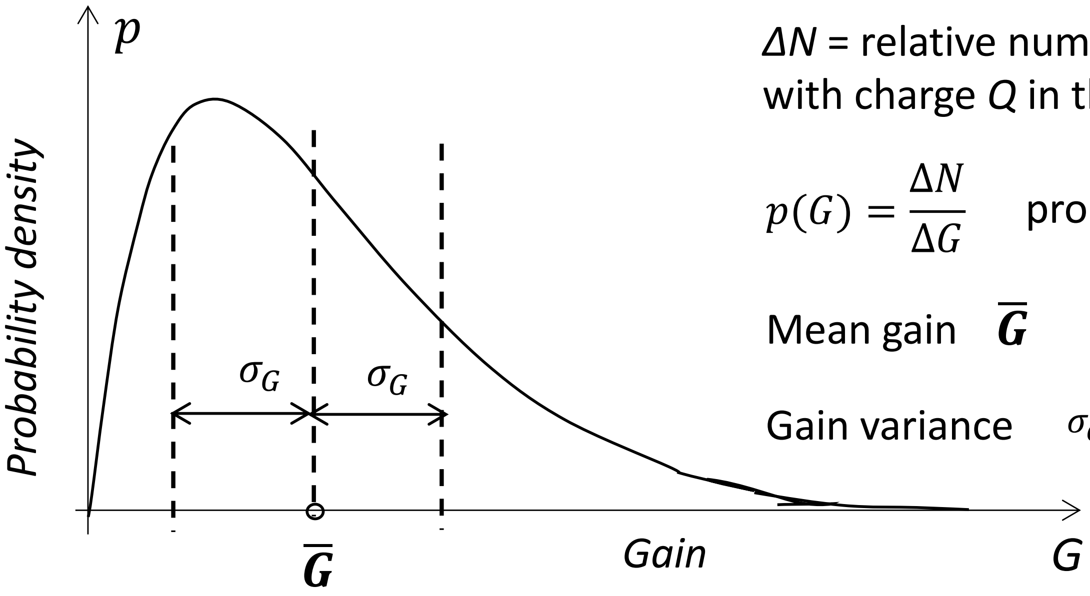
Single Electron Response SER

- The PMT output is superposition of elementary current pulses that correspond to single electrons emitted by the cathode, called **Single Electron Response (SER)** pulses.
- SER current pulses are fast (a few nanosecond width) and fairly high (pulse-charge Gq from 10^5 to 10^6 electrons). **They are remarkably higher than the noise** of fast circuits; with PMT weakly illuminated they are well observable on the oscilloscope screen and each of them corresponds to the detection of a single photon.
- The SER current pulses observed have all equal pulse shape, but **randomly varying pulse-amplitude**; i.e. G is not constant, but statistical



- The random fluctuations of G are due to the statistical nature of secondary electron emission
- Since the SER charge is much higher than the minimum measurable detector pulse, the statistical distribution *p(G)* of the gain *G* can be directly collected by measuring and classifying the pulse-charge of many SER pulses.

Statistical Distribution of the PMT gain



ΔN = relative number of SER pulses observed with charge *Q* in the interval *qG ± qΔG/2*

$p(G) = \frac{\Delta N}{\Delta G}$ probability density of *G*

Mean gain $\overline{\boldsymbol{G}}$

Gain variance $\sigma_G^2 = \overline{\left(G - \overline{G}\right)^2} = \overline{G^2} - \left(\overline{G}\right)^2$

- The plot above sketches the typical appearance of the statistical distribution *p(G)* of the PMT gain *G.*
- For different PMT models and different operating conditions (bias voltage distribution on dynodes; temperature of operation; etc.) remarkably different *p(G)* are observed. The distributions are roughly akin to gaussian, but skewed toward high *G* values.
- The main parameters to be considered for analyzing the PMT operation are mean gain $\bar{G}$, gain variance σ_G^2 and relative variance $v_G^2 = \frac{\sigma_G^2}{(\bar{G})^2}$

Excess Noise due to Gain Fluctuations

- Emission of primary electrons from cathode is a process with Poisson statistics, i.e. mean number N_p , variance $\sigma_p^2 = N_p$ and relative variance $v_p^2 = \frac{\sigma_p^2}{N_p^2} = \frac{1}{N_p}$
- Emission is followed in cascade by statistical multiplication with fluctuating G
- The mean of the cascade output is $N_u = N_p \cdot \bar{G}$ (two independent processes)
- The Laplace theory of probability generating functions shows that the relative variance $v_u{}^2$ of the output of a cascade is sum of the relative variance of every stage in the cascade divided by the mean value of all the previous stages. In our case:

$$v_u^2 = \frac{\sigma_u^2}{N_u^2} = v_p^2 + \frac{v_G^2}{N_p} = \frac{1}{N_p} + \frac{v_G^2}{N_p} = \frac{1}{N_p}(1 + v_G^2)$$

- The variance σ_u^2 thus is

$$\sigma_u^2 = N_p^2 \overline{G}^2 v_u^2 = N_p \overline{G}^2 (1 + v_G^2) = \sigma_p^2 \, \overline{G}^2 (1 + v_G^2)$$

In conclusion, the PMT :
1) amplifies the input variance by the square gain $\bar{G}^2$, like an amplifier and
2) further enhances it by the **Excess Noise Factor F** due to the gain fluctuations

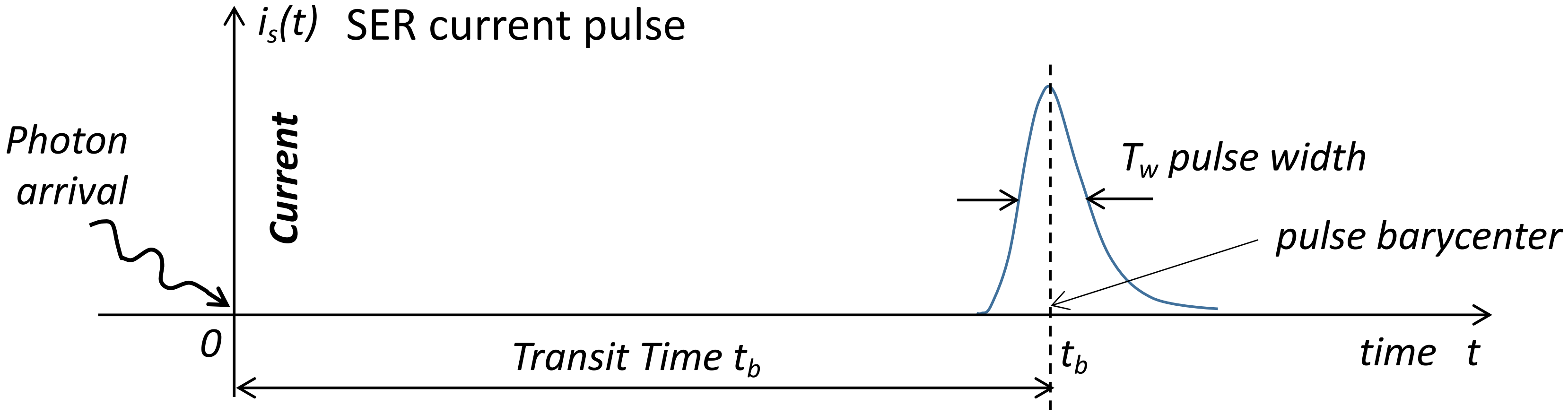
$$\boxed{\sigma_u^2 = \sigma_p^2 \cdot \overline{G}^2 \cdot F} \quad \text{with} \quad \boxed{F = 1 + v_G^2 > 1}$$

***F ≤ 2* for most PMT types and *F* is close to unity for high quality PMT types**.

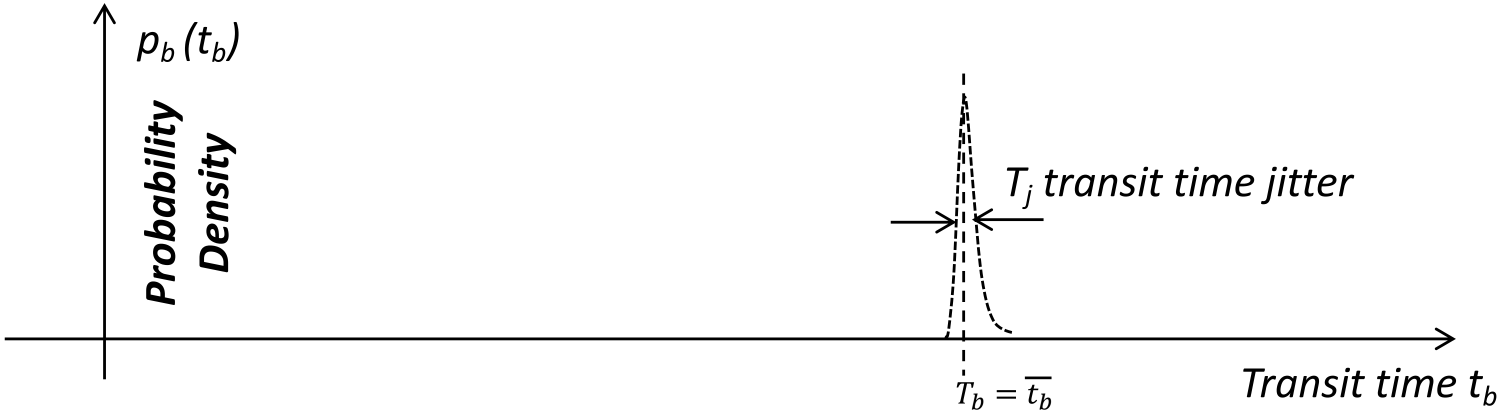
Dynamic response of PMTs

PMT Dynamic Response: SER pulse

PMT response to a single photon



Transit Time distribution

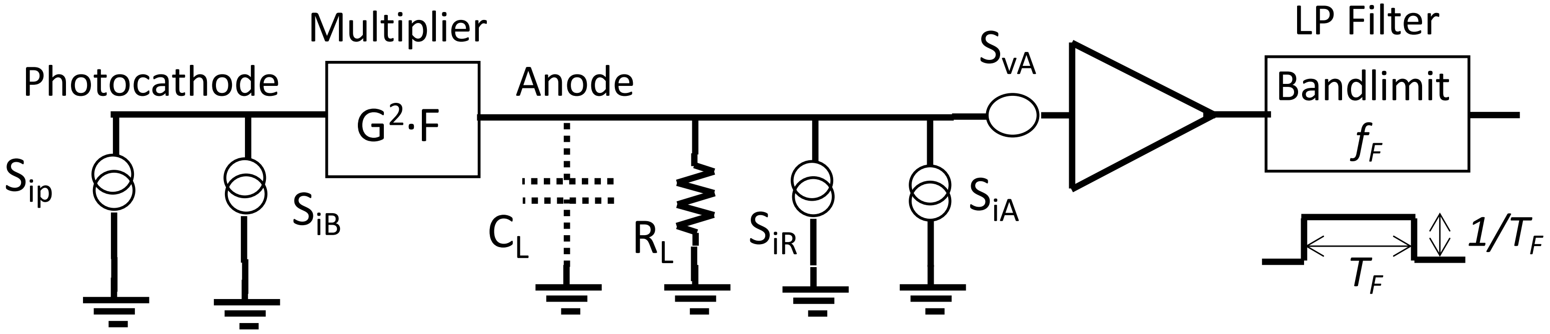


PMT Dynamic Response: SER pulses

- Differently from vacuum tube photodiodes, in PMT the rise of a SER current pulse is delayed (from ≈10ns to some 10ns dependent on PMT type and bias voltage) with respect to the photon arrival. The dynodes electrostatically screen the anode, so that only electrons traveling from last dynode to anode induce current (Shockley-Ramo theorem).
  The **PMT transit time t_b** is defined as the delay of the **pulse barycenter**.
- **The transit time t_b randomly fluctuates from pulse to pulse, with a transit time jitter T_j** (full-width at half maximum FWHM of the t_b distribution) from a few 100ps to a few ns depending on PMT type and bias voltage. T_j is due to the statistical dispersion of the electron trajectories in the *first stages of the multiplier*.
- **The SER pulse width T_w** (FWHM from a few ns to various ns, depending on PMT type and bias voltage) is always wider than the transit time jitter: $T_w \approx$ 5 to 10 times T_j . It is due to the statistical dispersion of the electron trajectories *in all the multiplier*.
- T_w has very small fluctuations, practically negligible

Signal-to-Noise Ratio and Minimum Measurable Signal

Noise sources and filtering with PMTs



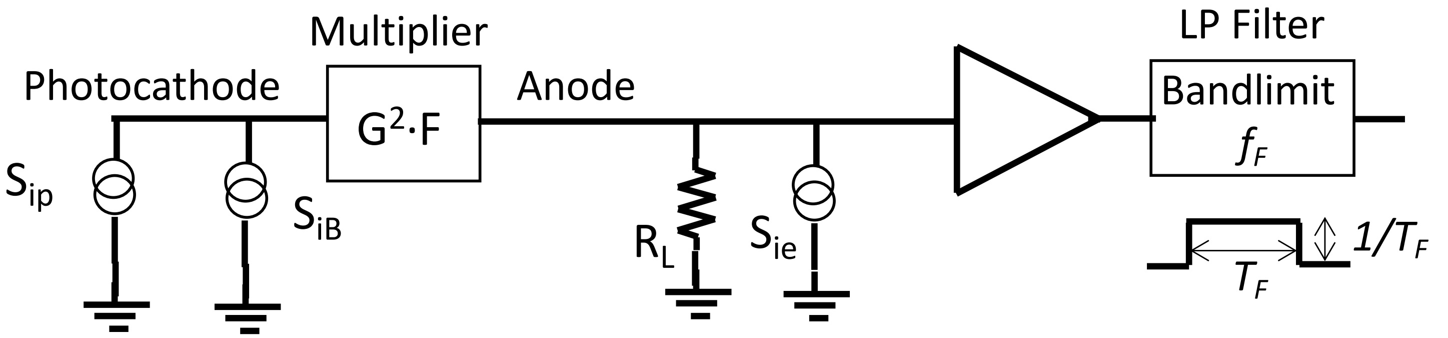
- n_p photoelectron rate → $I_p = n_p q$ photocurrent
- n_D dark electron rate → $I_D = n_D q$ cathode dark current
- n_b electron rate due to photon background → $I_b = n_b q$ photon background current
- $n_B = n_D + n_b$ total background electron rate → $I_B = n_B q$ total background current

Noise sources :

- at cathode: $S_{ip} = 2qI_p = 2q^2 n_p$ photocurrent noise, **increases with the signal**
- at cathode: $S_{iB} = 2qI_B = 2q^2 n_B$ background noise, **independent from the signal**
- at anode: resistor load noise S_{iR} and preamplifier noise S_{iA} and S_{iR}

Let’s deal with S/N and minimum measurable signal in the basic case:
constant signal current I_p and **low-pass filtering** (typically by Gated Integration)

Noise sources and filtering with PMTs



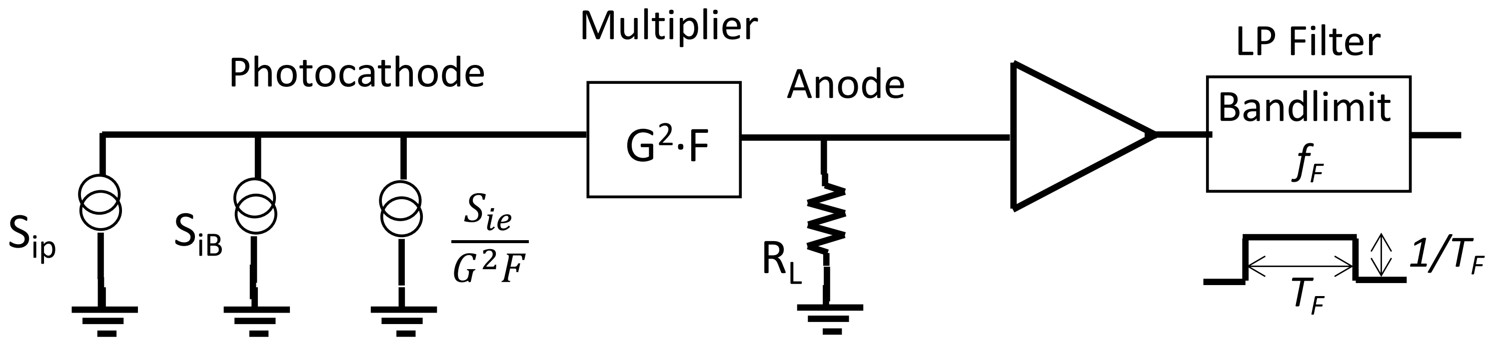
We consider cases with wide-band load, i.e. with $1/4R_LC_L >> f_F$, such that

a) the filtering effect of C_L is negligible

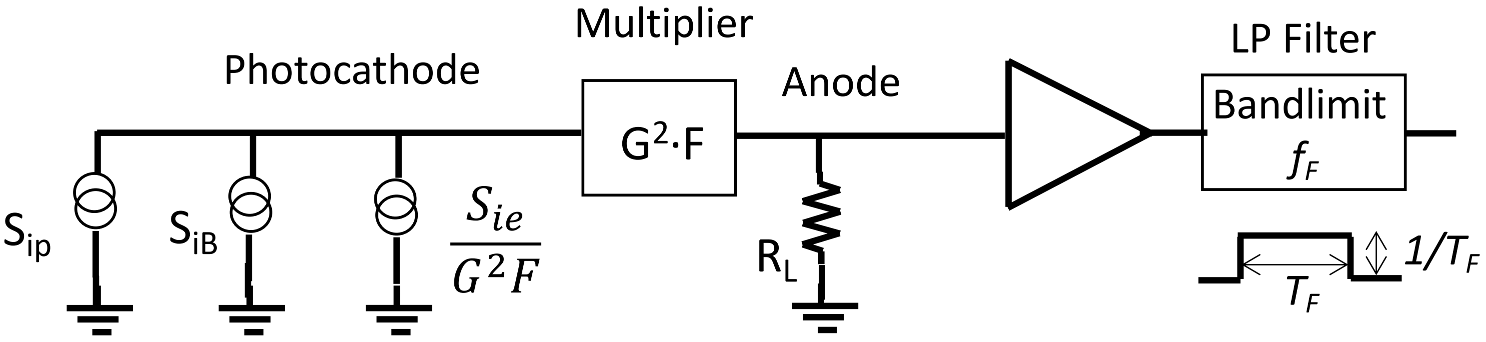
b) the circuit noise can modeled simply by a current generator

$$S_{ie} = S_{iA} + S_{iR} + \frac{S_{vA}}{R_L^2}$$

which can be referred back to the input (at the photocathode) as S_{ie}/G^2F



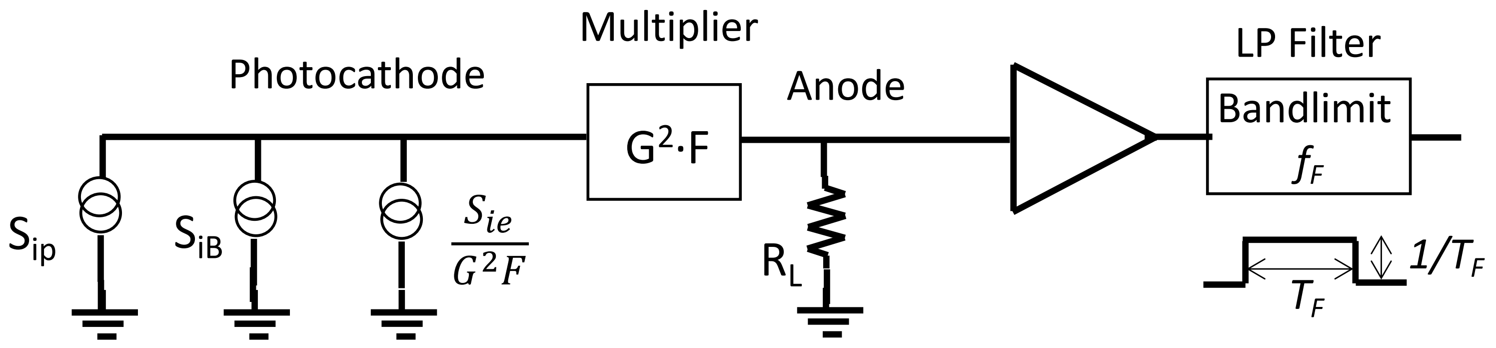
Role of the Circuit Noise



- The circuit noise S_{ie} can be modeled by a shot current **at the anode:**
  $I_e = S_{ie}/2q$ with electron rate $n_e = I_e/q = S_{ie}/2q^2$
- With wide band preamplifier and low resistance $R_L \approx$ few kΩ the circuit noise typically is $\sqrt{S_{ie}} \approx 2\, pA/\sqrt{Hz}$ or more. The equivalent shot electron rate is $\boldsymbol{n_e \approx 10^{14}}$ ***el/s*** **or more**
- Referred to input (cathode), the circuit noise is modeled by a shot current with **reduced** electron rate n_e/FG^2. For instance, with G= 10^6 it is $\boldsymbol{n_e/FG^2 \approx 100}$ ***el/s***
- The circuit noise referred to the input added to the background noise $S_{iB}=2qI_B=2q^2n_B$ gives the **constant** noise component (i.e. **NOT** dependent on the signal)

$$S_{iB} + \frac{S_{ie}}{G^2F} = 2qI_B + \frac{2qI_e}{G^2F} = 2q^2\left(n_B + \frac{n_e}{G^2F}\right)$$

Role of the Circuit Noise



$$S_{iB} + \frac{S_{ie}}{G^2 F} = 2qI_B + \frac{2qI_e}{G^2 F} = 2q^2 \left(n_B + \frac{n_e}{G^2 F}\right)$$

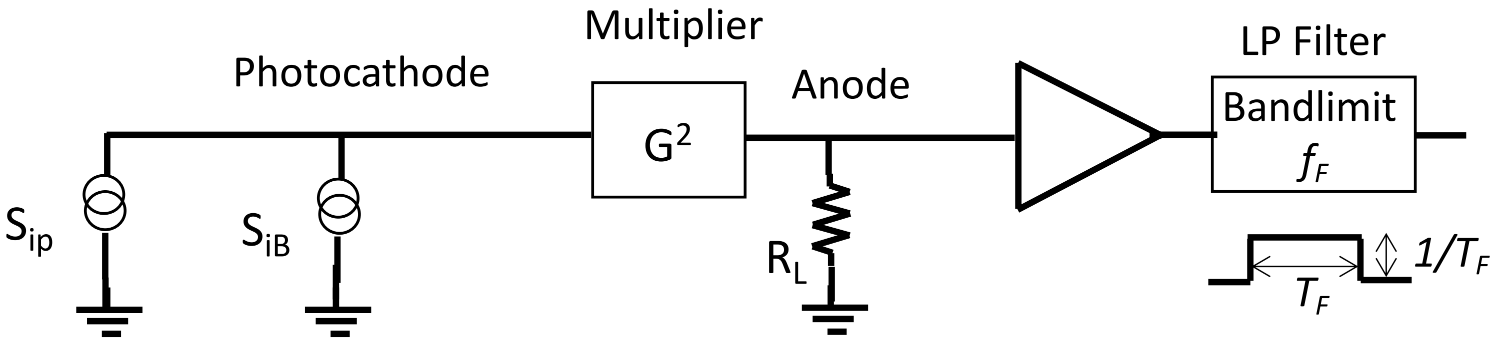
- The role of the circuit noise is assessed by comparing it to the constant noise source of the PMT, the background noise $S_{iB}=2qI_B=2q^2 n_B$
- The background electron rate at the cathode n_B may vary ***from a few el/s to a few 10^6 el/s***, depending on the photocathode type and operating temperature and on the background light level (see Slides PD2)
- In **most cases** of PMT application it is $n_B >> n_e/G^2F$: the equivalent electron rate n_e/G^2F is **totally negligible** with respect to n_B , the circuit noise plays no role
- In cases with moderate gain G and/or very low dark current the circuit noise contribution may be significant and is very simply taken into account, by employing the resulting density of constant noise component in the evaluation

Minimum Measurable Signal

For the sake of simplicity in the following computations we consider:

a) **negligible circuit noise.** Anyway, we know when it must be taken into account and how to do it, by considering an increased constant component of noise.

b) **negligible excess noise, i.e. *F = 1*.** Anyway, cases with non-negligible F > 1 can be taken into account simply by introducing the factor $\sqrt{F}$ to decrease the S/N and increase the noise variance and the minimum signal computed with F=1.



$$\boxed{\frac{S}{N} = \frac{I_p}{\sqrt{S_{ip}f_F + S_{iB}f_F}} = \frac{I_p}{\sqrt{2qI_pf_F + 2qI_Bf_F}}}$$

The minimum signal $I_{p,min}$ is reached when S/N = 1 : we will see that the result markedly depends on the **relative size of constant noise vs photocurrent noise**

Minimum Signal limited by Photocurrent-Noise

- The simplest **extreme case is with negligible background noise:** only photocurrent noise matters. With noise band-limit $f_F = 1/2T_F$ (GI filtering)

$$\frac{S}{N} = \frac{I_p}{\sqrt{2qI_p f_F}} = \frac{I_p T_F}{\sqrt{qI_p T_F}} = \sqrt{\frac{I_p T_F}{q}} = \sqrt{n_p T_F} = \sqrt{N_p}$$

- $N_p = n_p T_F$ is the **number of photoelectrons** in the filtering time T_F .
- In fact, the S/N can be obtained directly from the Poisson statistics of photoelectrons: with mean number N_p , the variance is $\sigma_p^2 = N_p$ and

$$\frac{S}{N} = \frac{N_p}{\sigma_p} = \frac{N_p}{\sqrt{N_p}} = \sqrt{N_p}$$

- Remark that in this case the noise is **NOT constant,** independent from the signal: as the signal goes down, **also the noise goes down!!**

Minimum Signal limited by Photocurrent-Noise

- By making lower and lower I_p , **when S/N = 1 the minimum signal $I_{p,min\text{-}p}$ is reached**

$$\boxed{\left(\frac{S}{N}\right)_{min} = 1 = \sqrt{\frac{I_{p,\min -p} T_F}{q}} = \sqrt{n_{p,\min -p}\ T_F} \sqrt{N_{p,\min -p}}}$$

- The minimum measurable photocurrent signal $I_{p,min\text{-}p}$ corresponds to just **one photoelectron in T_F** , the filter weighting time:

$$\boxed{I_{p,\min -p} = \frac{q}{T_F}} \qquad \boxed{n_{p,\min -p} = \frac{1}{T_F}} \qquad \boxed{N_{p,\min -p} = 1}$$

- Observing the complete S/N equation

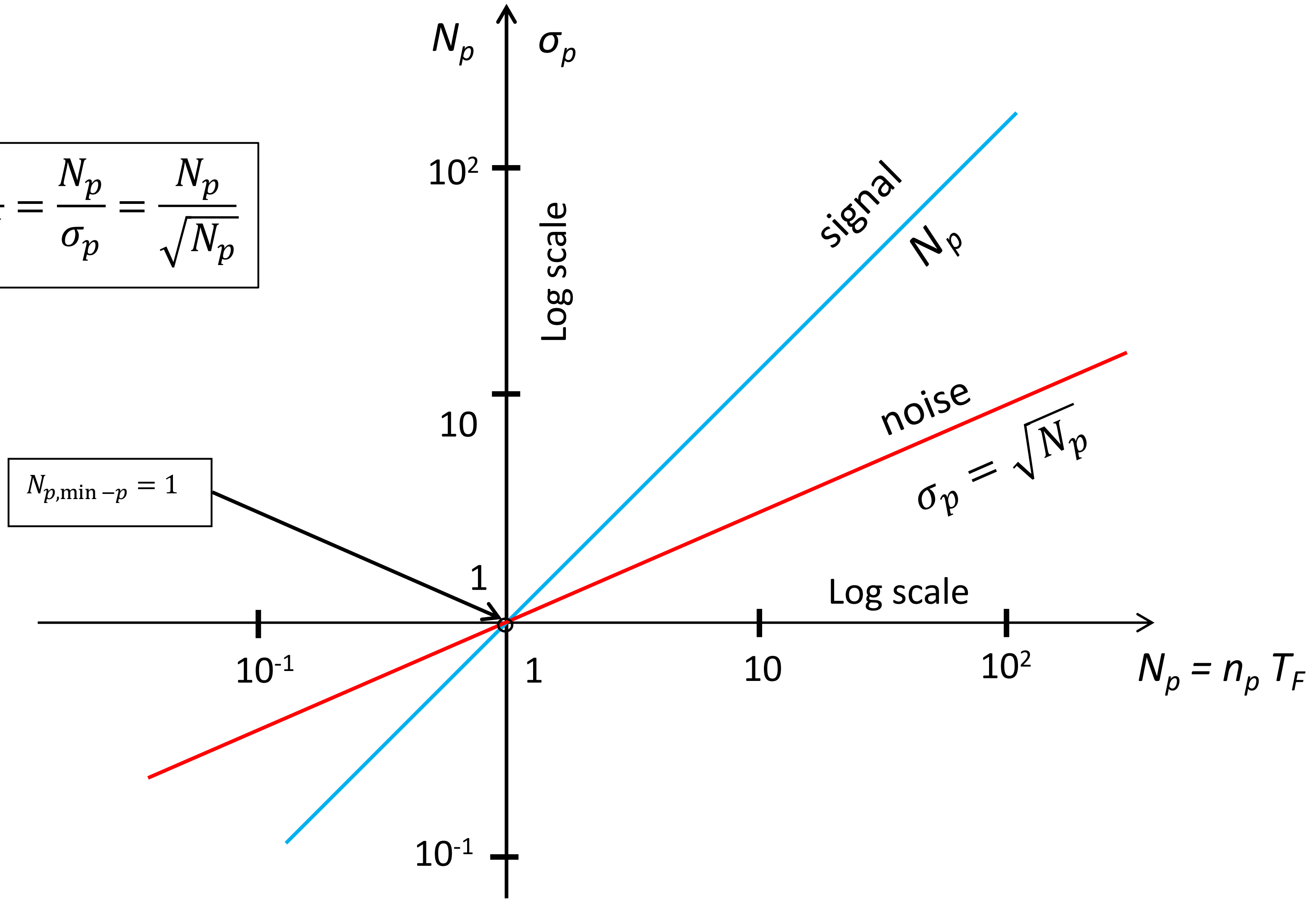
$$\frac{S}{N} = \frac{I_p}{\sqrt{2qI_p f_F + 2qI_B f_F}} = \frac{I_p T_F}{\sqrt{qI_p T_F + qI_B T_F}} = \frac{n_p T_F}{\sqrt{n_p T_F + n_B T_F}} = \frac{N_p}{\sqrt{N_p + N_B}}$$

we see that the background noise is truly negligible only if $I_B << I_p$ for any I_p **down to the minimum $I_{p,\ min\text{-}p}$** , i.e. only if

$$\boxed{I_B \ll \frac{q}{T_F}} \qquad \boxed{n_B \ll \frac{1}{T_F}} \qquad \boxed{N_B \ll 1}$$

Minimum Signal limited by Photocurrent-Noise

$$\frac{S}{N} = \frac{N_p}{\sigma_p} = \frac{N_p}{\sqrt{N_p}}$$



Signal measured by **charge**, in terms of **number of photoelectrons $N_p = n_p T_F$**

Minimum Signal limited by Background Noise

- The **opposite extreme case is with negligible photocurrent noise:** only background noise matters. More precisely, it’s the case where the limit current $I_p = I_{p,min-p}$ computed with only the photocurrent noise is much lower than the background current I_B

$$\boxed{I_B \gg \frac{q}{T_F}} \qquad \boxed{n_B \gg \frac{1}{T_F}} \qquad \boxed{N_B \gg 1}$$

- There is now a different **minimum signal $I_{p,min-B}$ limited by the background noise**

$$\boxed{I_{p,\min -B} = \sqrt{\frac{qI_B}{T_F}}} \qquad \boxed{n_{p,\min -B} = \sqrt{\frac{n_B}{T_F}}} \qquad \boxed{N_{p,\min -B} = \sqrt{N_B}}$$

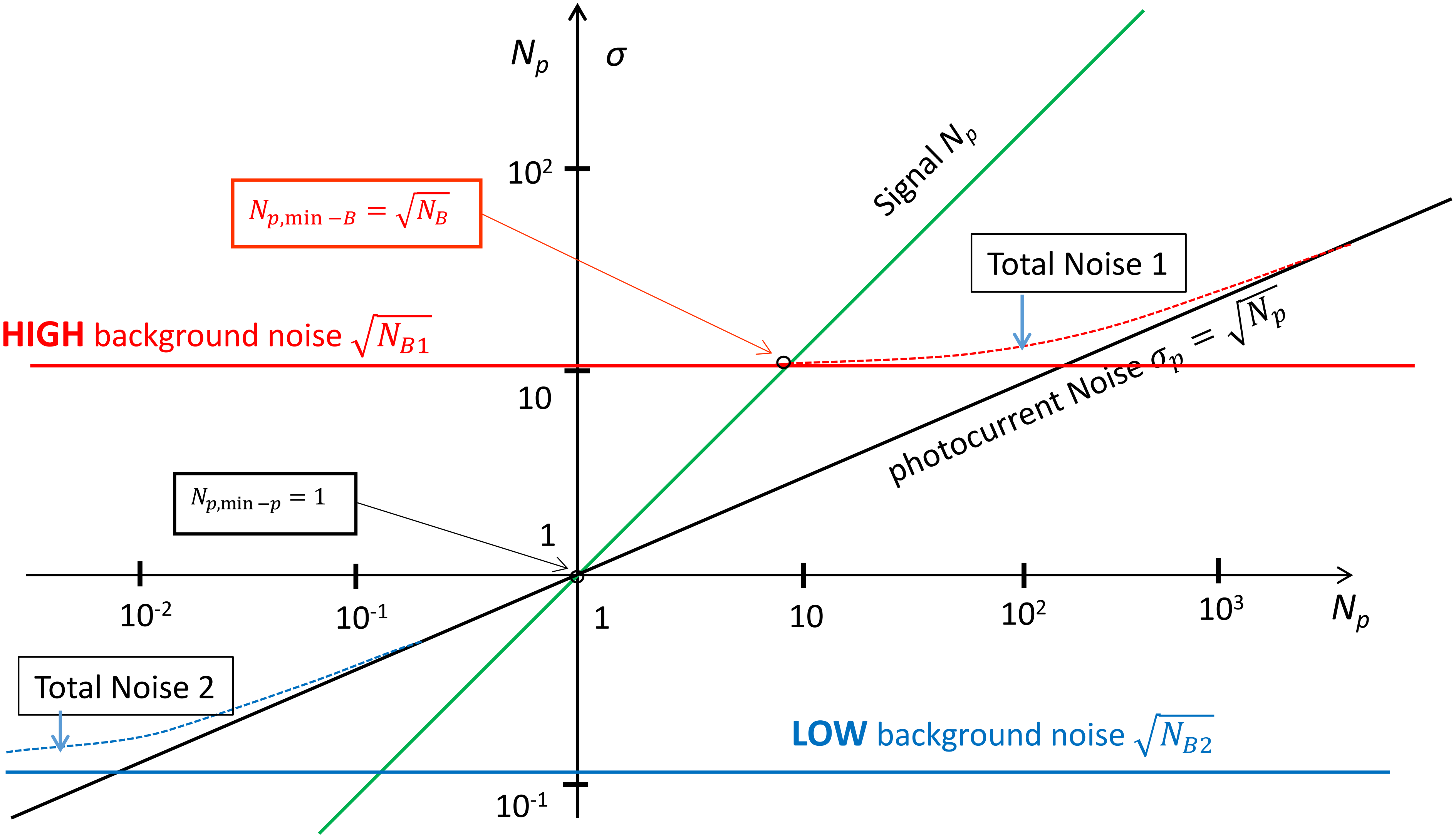
- In **intermediate cases both noise components** contribute to limit the minimum signal, which is computed from

$$\frac{S}{N} = \frac{N_{p,\min}}{\sqrt{NB_{p,\min}}} = 1$$

2nd order equation that leads to

$$N_{p,\min} = \frac{1}{2}\left(1 + \sqrt{1 + 4N_B}\right).$$

Minimum Signal limited by Noise



Signal **charge**, in terms of **number of photoelectrons $N_p = n_p T_F$**