COURSE OUTLINE

- Introduction
- Signals and Noise
- Filtering: Band-Pass Filters 3 – BPF3
- Sensors and associated electronics

Band-Pass Filtering 3

- Asynchronous Measurement of Sinusoidal Signals
- Principle of Synchronous Measurements of Sinusoidal Signals
- Noise Filtering in Synchronous Measurements
- Lock-in Amplifier Principle and Weighting Function

Asynchronous Measurement of Sinusoidal Signals

Asynchronous measurement of sinusoidal signals

- **Asynchronous** (or phase-insensitive) techniques were devised for measuring a sinusoidal signal without needing an auxiliary **reference** that points out the peaking time (i.e. the phase of the signal).

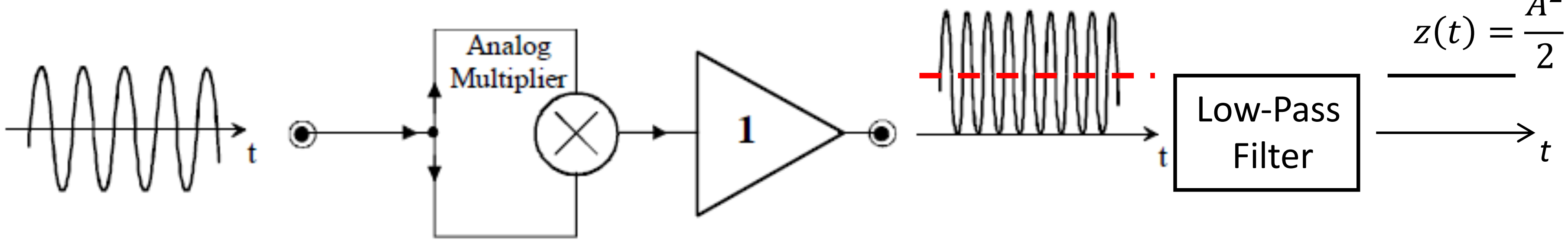
- They are currently employed in **AC voltmeters and amperometers**.

- The basic circuits of such meters are
  - the mean-square detector
  - the half-wave rectifier
  - the full-wave rectifier

- For a correct measurement of the amplitude of the sinusoidal signal, it is necessary to avoid feeding a DC component to the input of an asynchronous meter circuit. Therefore, the meter must be preceded by a filter that cuts off the low-frequencies, that is, a band-pass or a high-pass filter.

Asynchronous measurement of sinusoidal signals with Mean-Square Detector

$$x(t) = A\cos(\omega t + \vartheta) \qquad y(t) = A^2\cos^2(\omega t + \vartheta) = \frac{A^2}{2} + \frac{A^2}{2}\cos(2\omega t + 2\vartheta)$$

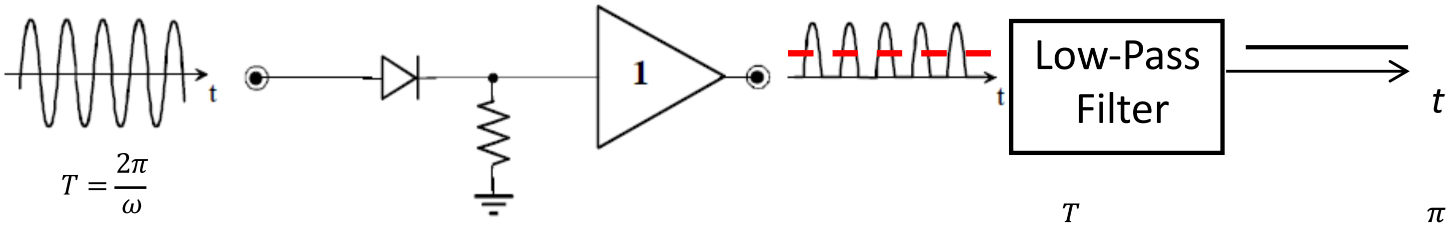


- It is a power-meter: the output is a measure of the **total** input mean power, sum of signal power (proportional to the square of amplitude A^2) plus noise power.
- The low-pass filter has NO EFFECT OF NOISE REDUCTION. In fact, it does not average the input, it averages **the square** of the input.
- For improving the S/N it is necessary to insert a filter **before** the Mean-Square Detector

Asynchronous measurement of sinusoidal signals with Rectifier

Half-Wave Rectifier (HWR)

$$x(t) = A\cos(\omega t)$$

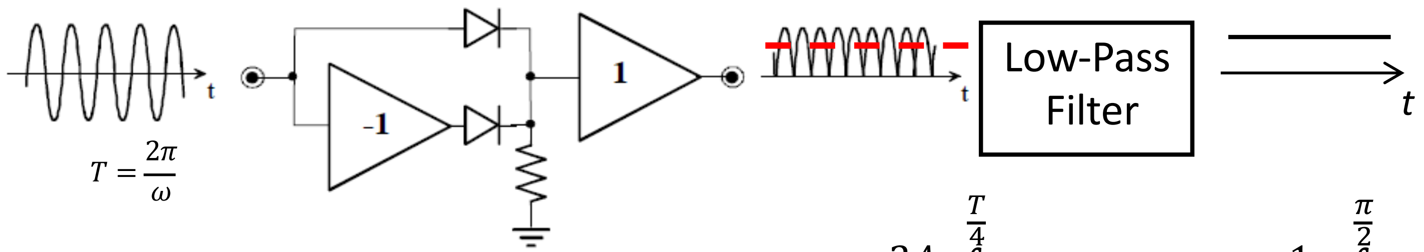


$$z(t) = \frac{A}{\pi}$$

$$z = \frac{A}{T}\int_{-\frac{T}{4}}^{\frac{T}{4}} \cos(\omega t)\,dt = A\frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\varphi\,d\varphi = \frac{A}{\pi}$$

Full-Wave Rectifier (FWR)

$$x(t) = A\cos(\omega t)$$



$$z(t) = \frac{2A}{\pi}$$

$$z = \frac{2A}{T}\int_{-\frac{T}{4}}^{\frac{T}{4}} \cos(\omega t)\,dt = 2A\frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\varphi\,d\varphi = \frac{2A}{\pi}$$

Asynchronous measurement of sinusoidal signals with Rectifier

- The measurement with a rectifier is not really asynchronous, it is **self-synchronized**. The sinusoidal signal itself decides when it has to be passed with positive polarity and when passed with negative polarity (in the full-wave rectifier) or not passed at all (in the half-wave rectifier).
- In such operation, the LPF reduces the contribution of the wide-band noise, thus improving the output S/N. However, this is true only if the input signal is remarkably higher than the noise, i.e. if the input S/N is high.
- **As the input signal is reduced the noise gains increasing influence on the switching time of the rectifier, which progressively loses synchronism with the signal and tends to be synchronized with the zero-crossings of the noise.**
- The loss of synchronization progressively degrades the noise reduction by the LPF. With moderate S/N the improvement due to LPF is modest; with low S/N it is very weak. With S/N < 1 there is no improvement, there is not even a measure of the signal: the output is a measure of the noise mean absolute value.
- **In conclusion, meters based on rectifiers can just improve an already good S/N. They can't help to improve a modest S/N and it is out of the question to use them when S/N <1. For improving S/N it is necessary to employ filters before the meter.**

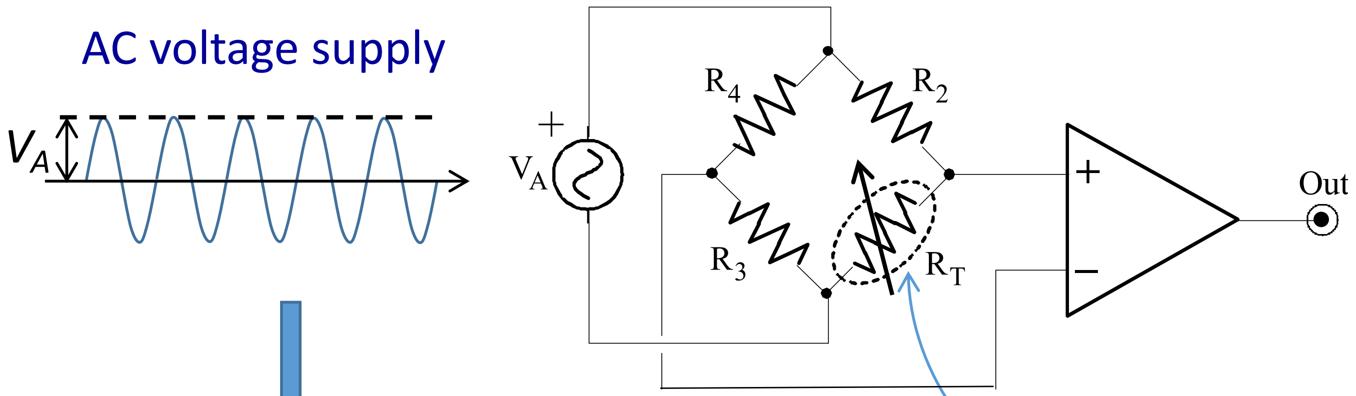
Synchronous (or Phase-Sensitive) Measurements of Sinusoidal Signals

Signal and Reference for Synchronization

KEY EXAMPLE

for the study of synchronous measurements and narrow-band filtering

AC voltage supply



SIGNAL OUT

$x(t) = A\cos(\omega_m t + \varphi)$

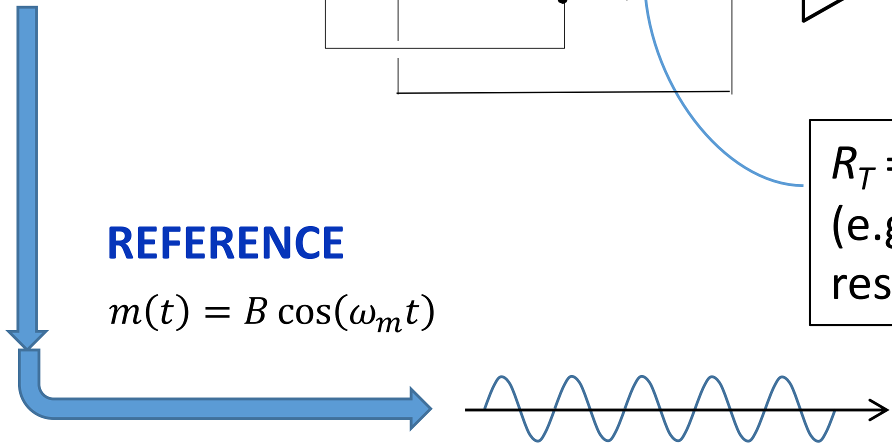
A to be measured

φ constant and known*

$R_T = R_T(\vartheta)$ resistance tracks a variable ϑ, (e.g. a temperature or a strain); resistances R_2 , R_3 , R_4 are constant

REFERENCE

$m(t) = B\cos(\omega_m t)$



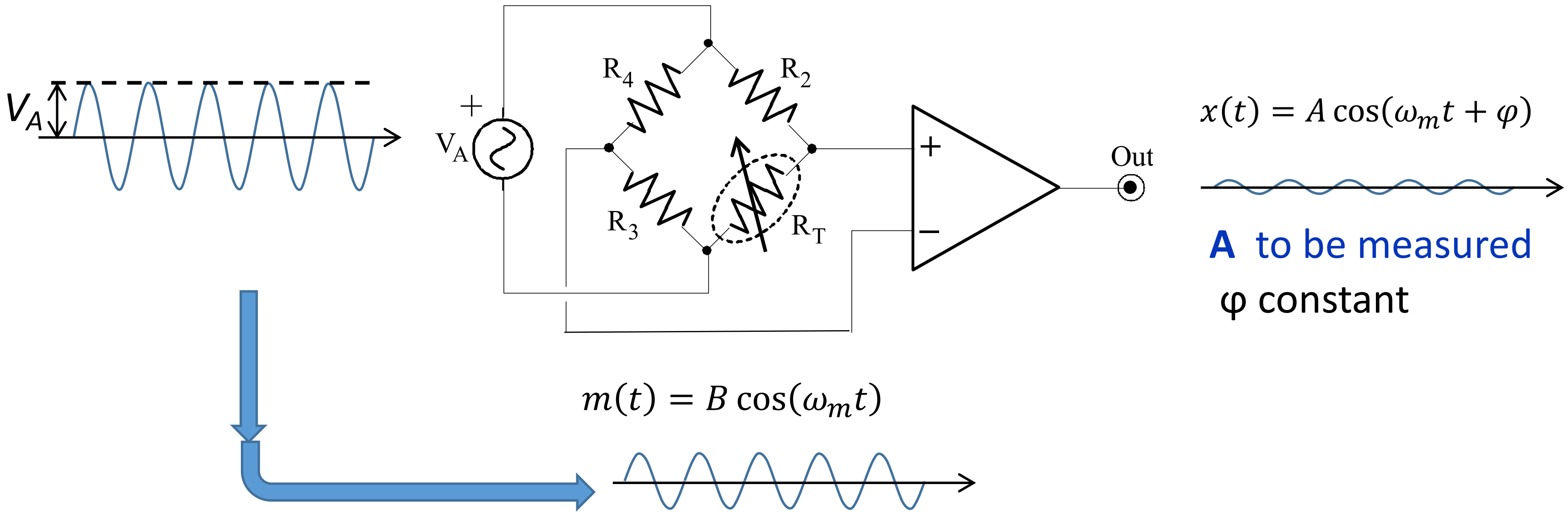
Shows the frequency and phase of the signal
i.e. points out the peak instants of the signal

* in this example φ=0 since the preamp passband limit is much higher than the signal frequency f_m

Signal and Reference for Synchronization

KEY EXAMPLE

for the study of synchronous measurements and narrow-band filtering



$x(t) = A\cos(\omega_m t + \varphi)$

A to be measured

φ constant

$m(t) = B\cos(\omega_m t)$

R_T e.g. strain sensor, the resistance varies following a mechanical **strain** $\boldsymbol{\vartheta}$

a) in cases with **constant** strain ϑ

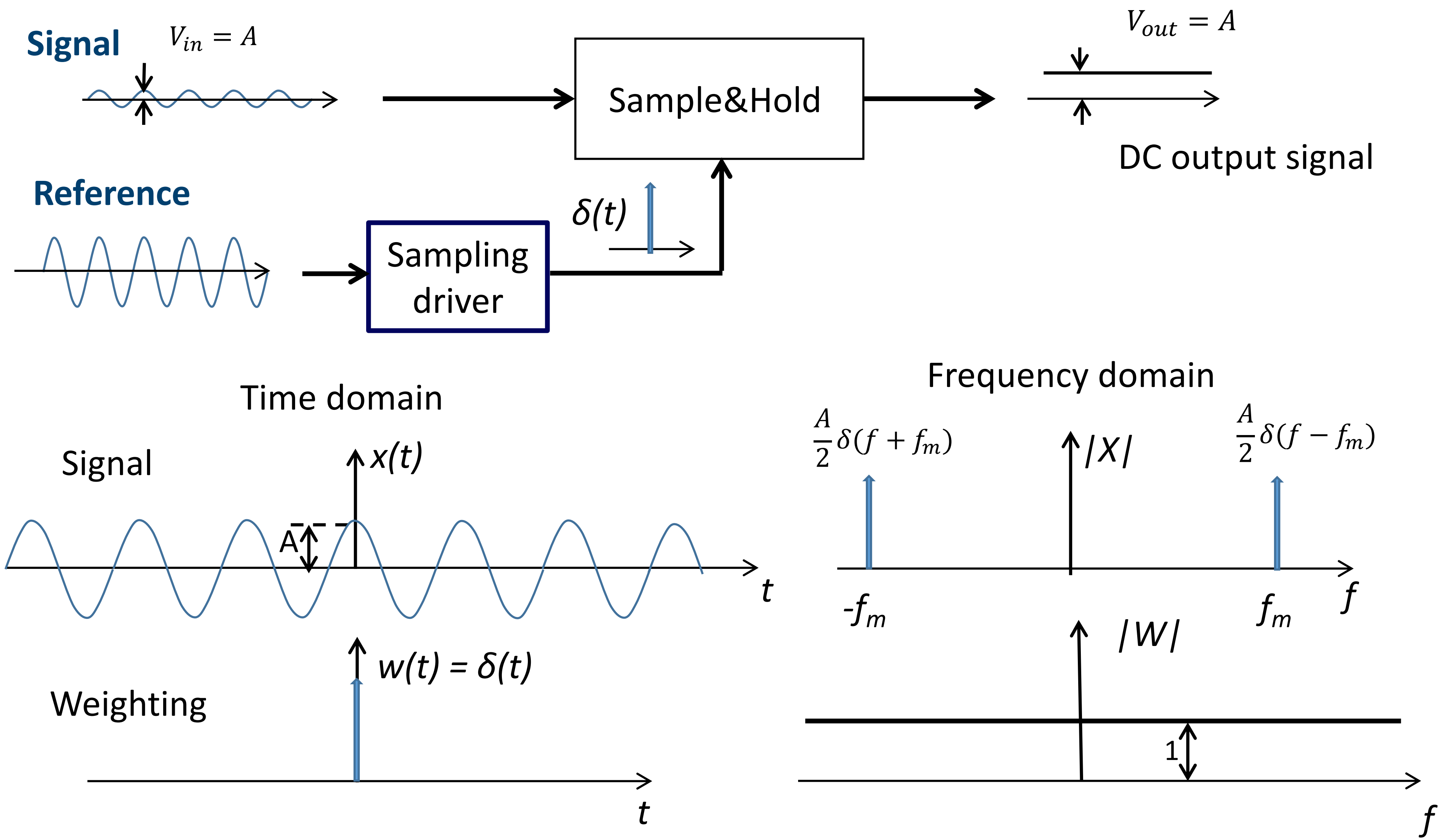
constant *A* → x(t) is a **pure sinusoidal** signal

b) in cases with **slowly variable** strain $\vartheta = \vartheta(t)$

variable $A = A(t)$ → x(t) is a **modulated sinusoidal** signal

SLOW variations = the Fourier components of $A(f)=F[A(t)]$ have frequencies $f << f_m$

Elementary synchronous measurement: peak sampling



NO FILTERING ACTION: output noise power = full input noise power

Noise Filtering in Synchronous Measurements

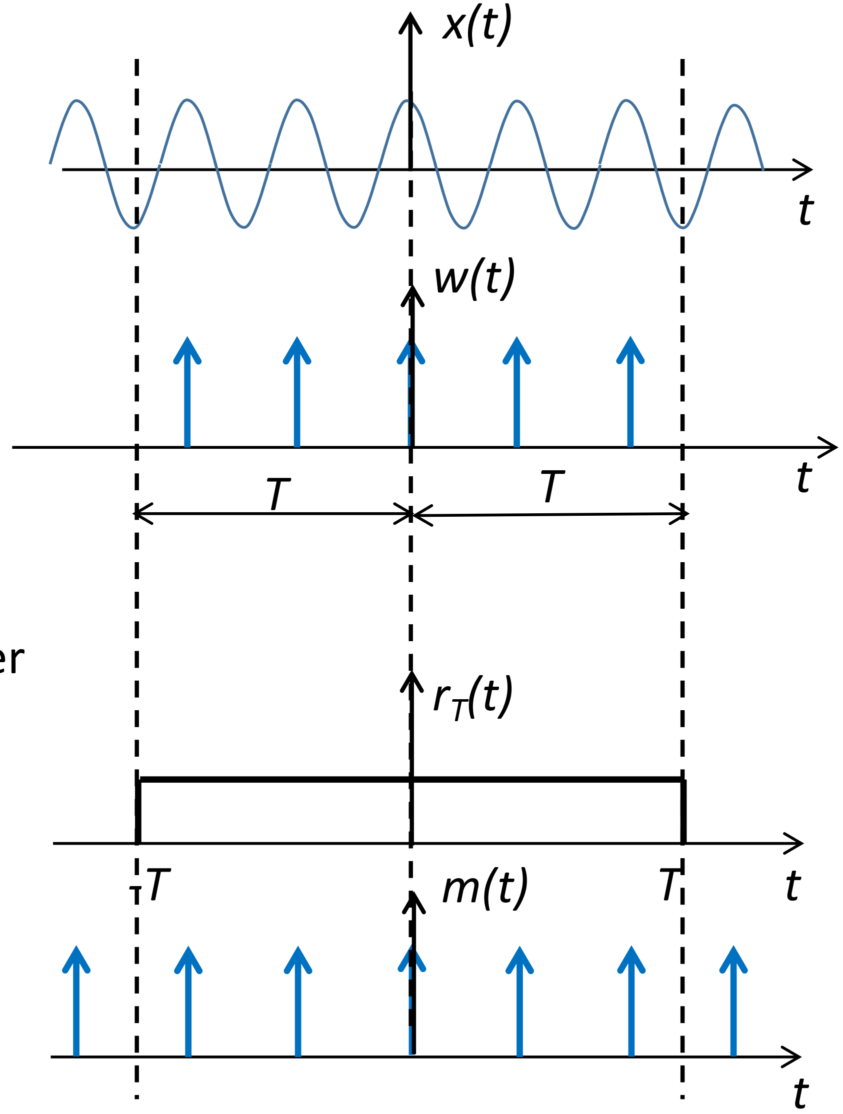
Synchronous measurement with averaging over many samples N >>1 of the peak

$$x(t) = A\cos(2\pi f_m t)$$

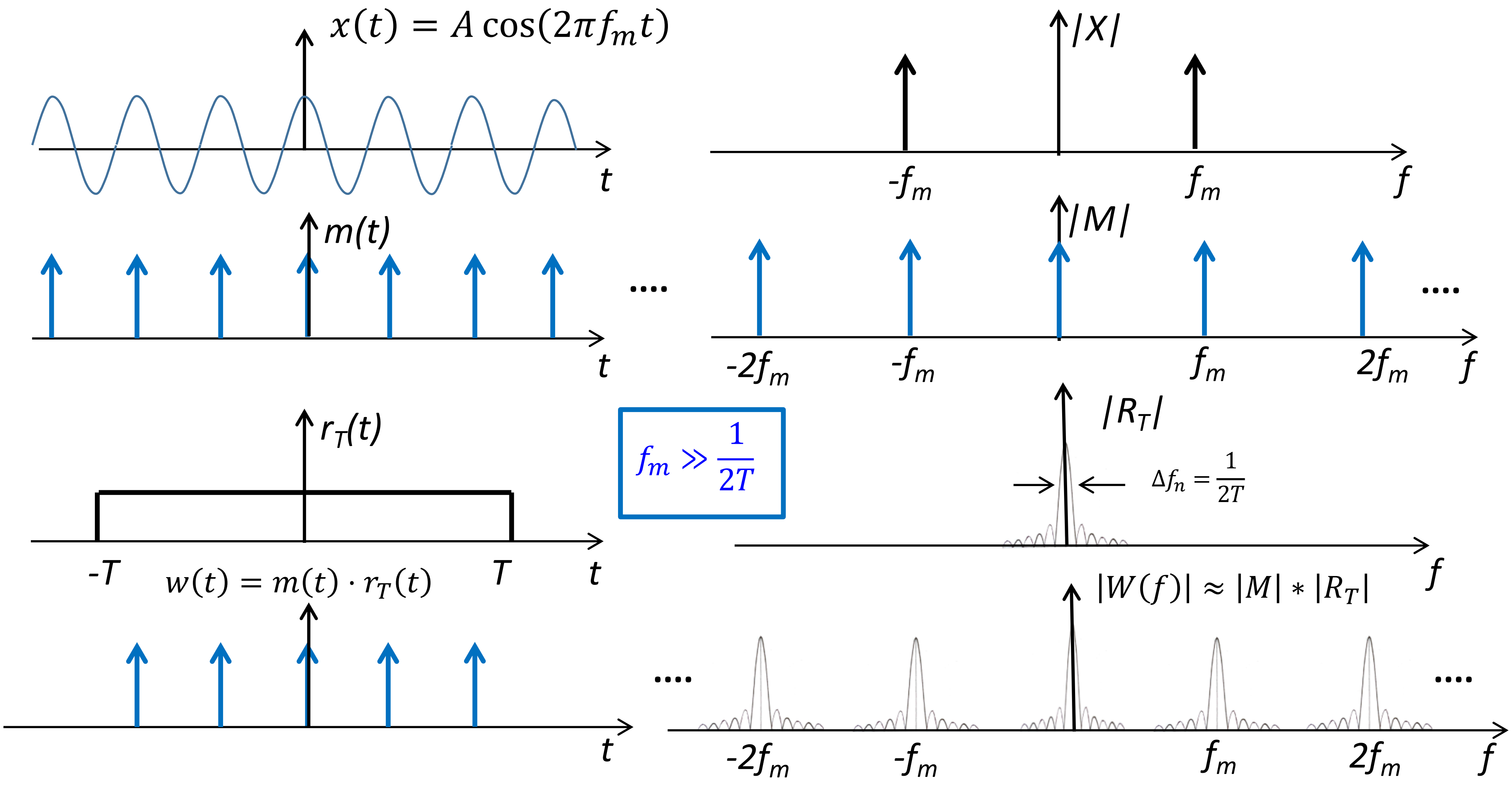
$$N = f_m 2T \gg 1$$

To take N samples
is equivalent to gate
a free-running sampler

$$w(t) = m(t) \cdot r_T(t)$$

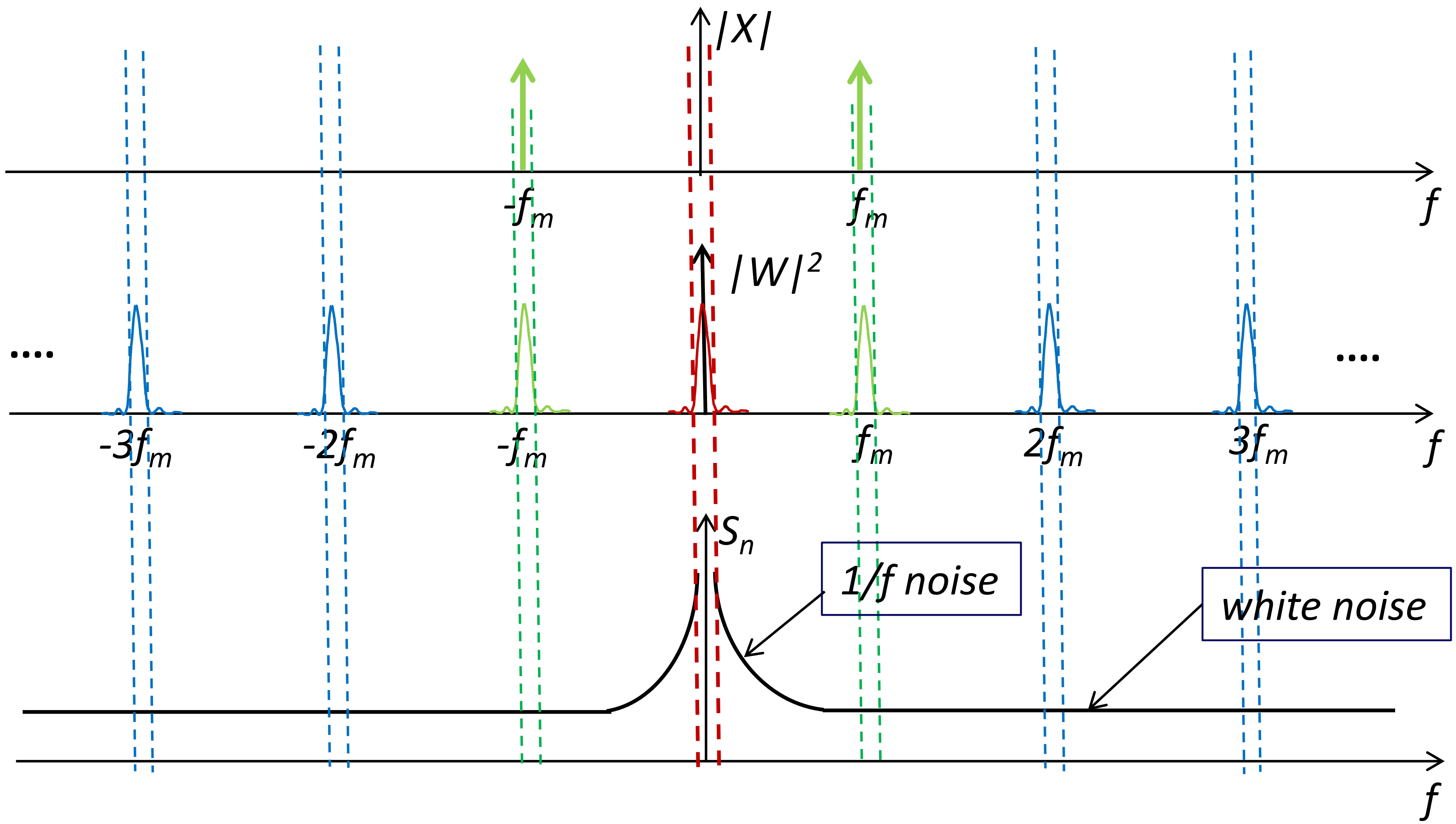


Synchronous measurement with averaging over many samples N >>1 of the peak



FILTERING: narrow bands at frequencies 0 , f_m , $2f_m$, $3f_m$ ....

Poor Noise Filtering by Sample-Averaging

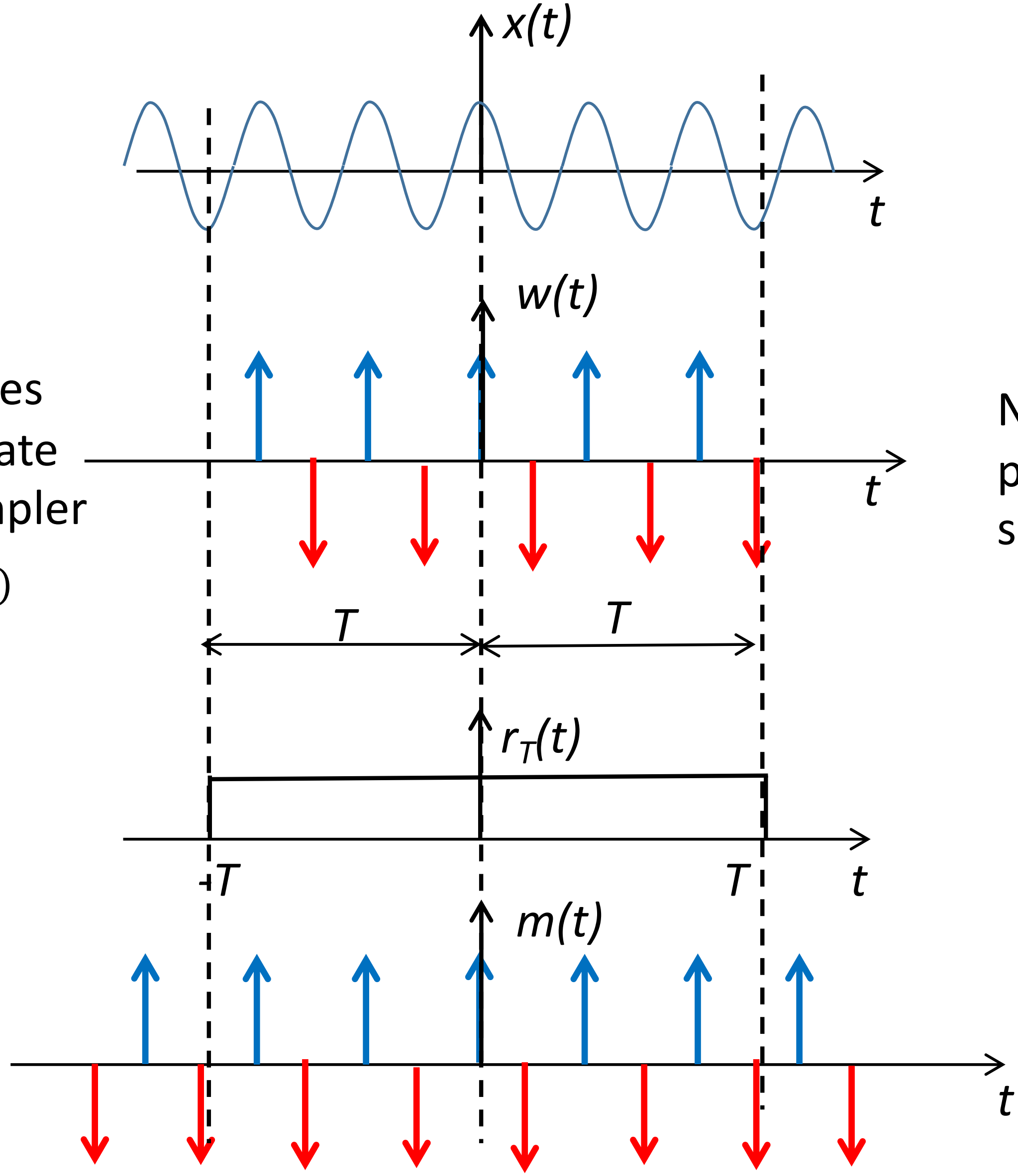


- At f_m **useful** band: it collects the **signal** and some white noise around it
- at f = 0 **VERY HARMFUL** band: it collects **1/f noise and no signal**
- at $2f_m$, $3f_m$, ... **harmful** bands: they collect just white noise without any signal

Synchronous measurement with DC suppression by summing positive peak and subtracting negative peak samples

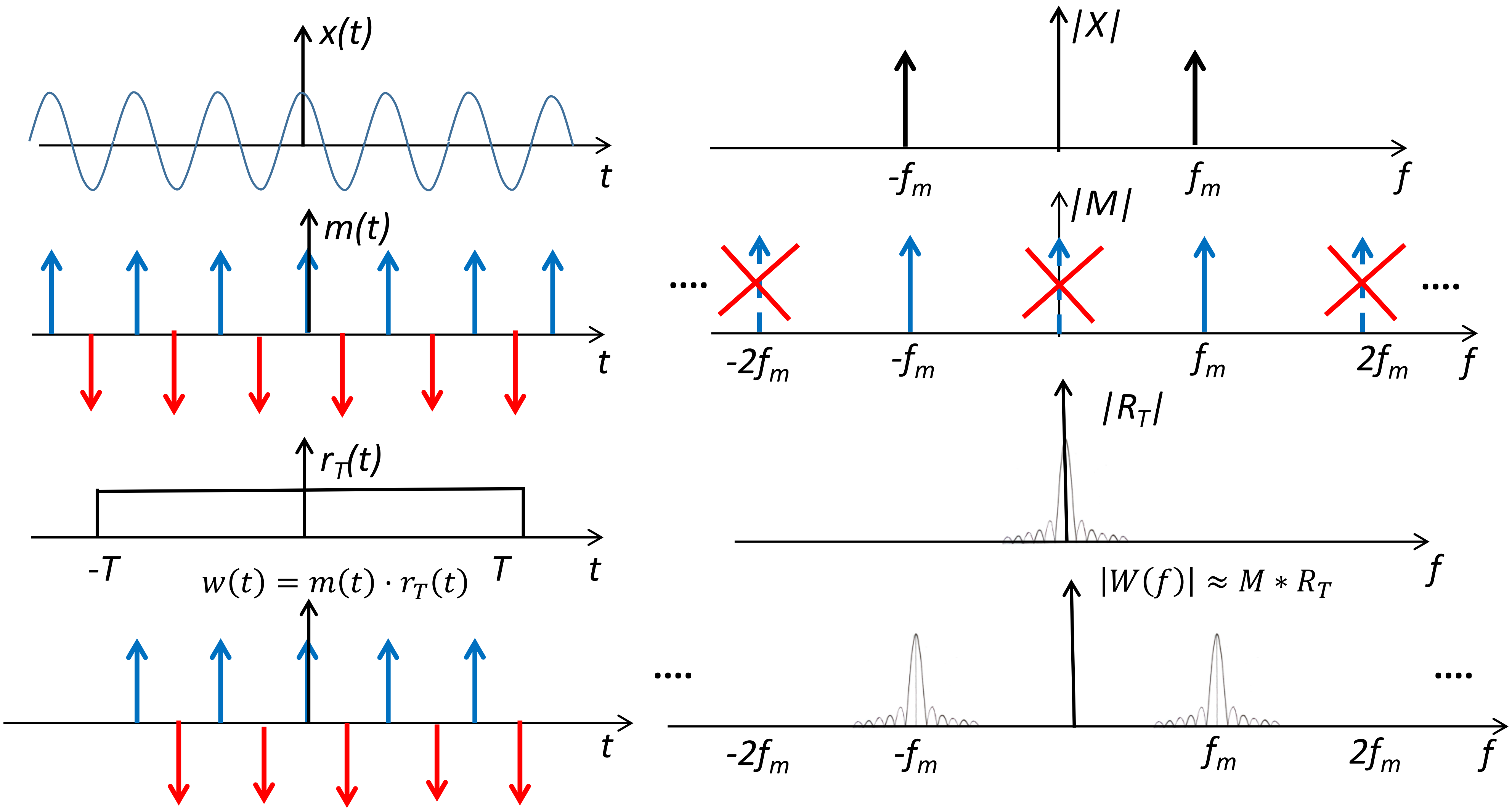
To take 2N samples is equivalent to gate a free-running sampler

$$w(t) = m(t) \cdot r_T(t)$$



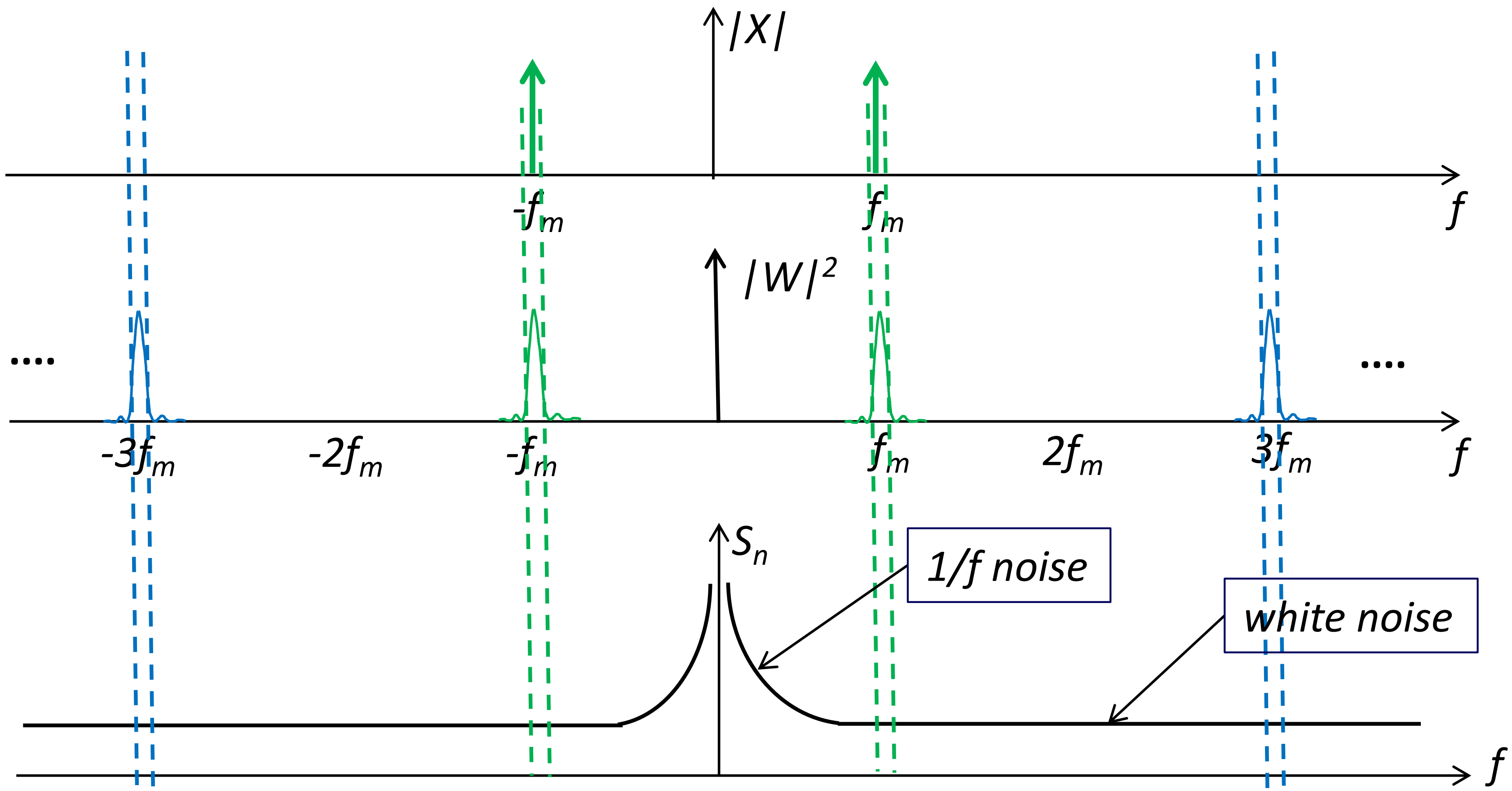
NB: equal number of positive and negative samples (zero net area)

Synchronous measurement with DC suppression by summing positive peak and subtracting negative peak samples



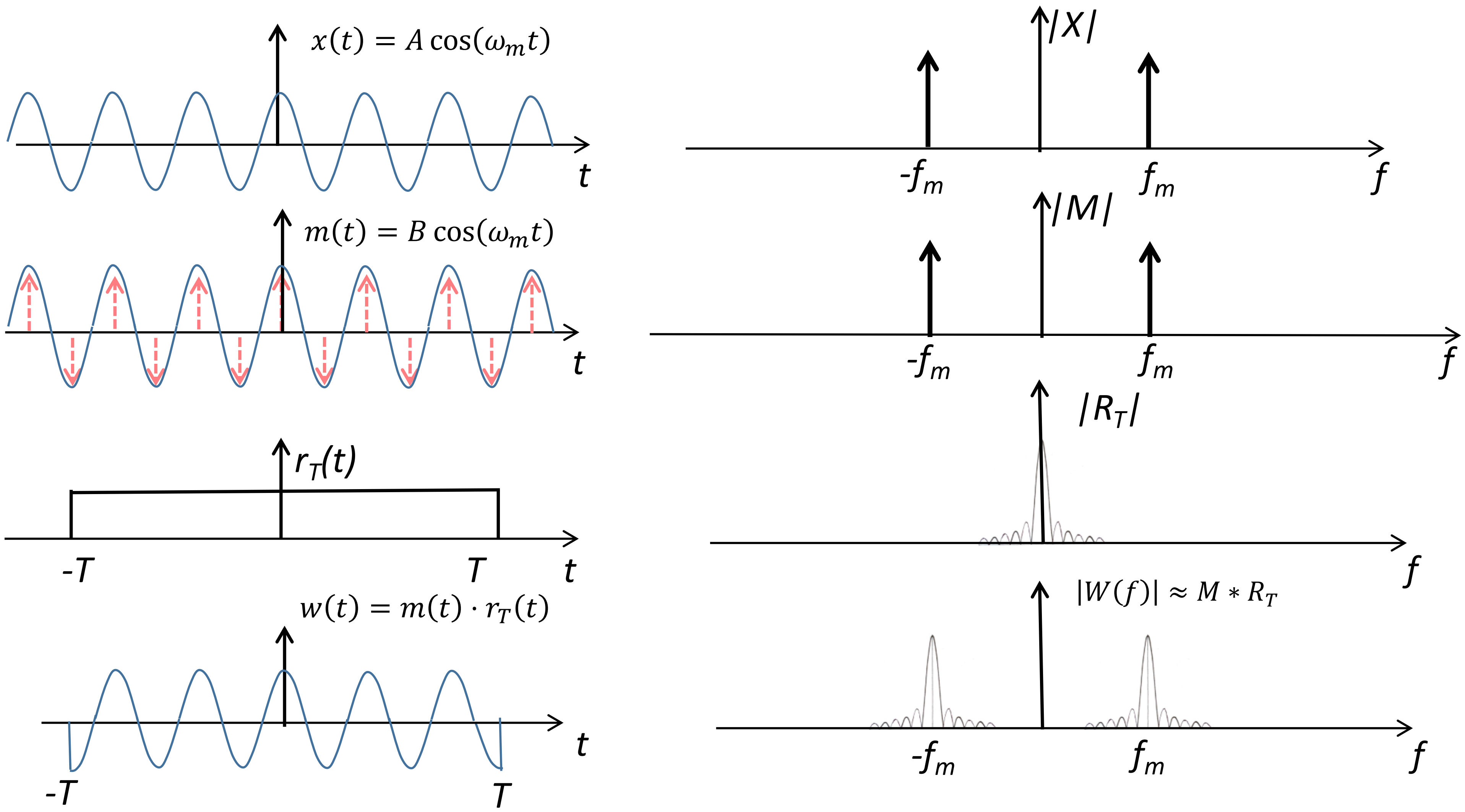
FILTERING : narrow bands at f_m and at **odd** multiples $3f_m$, $5f_m$ ....

Improved Filtering by Sample-Averaging with DC Suppression



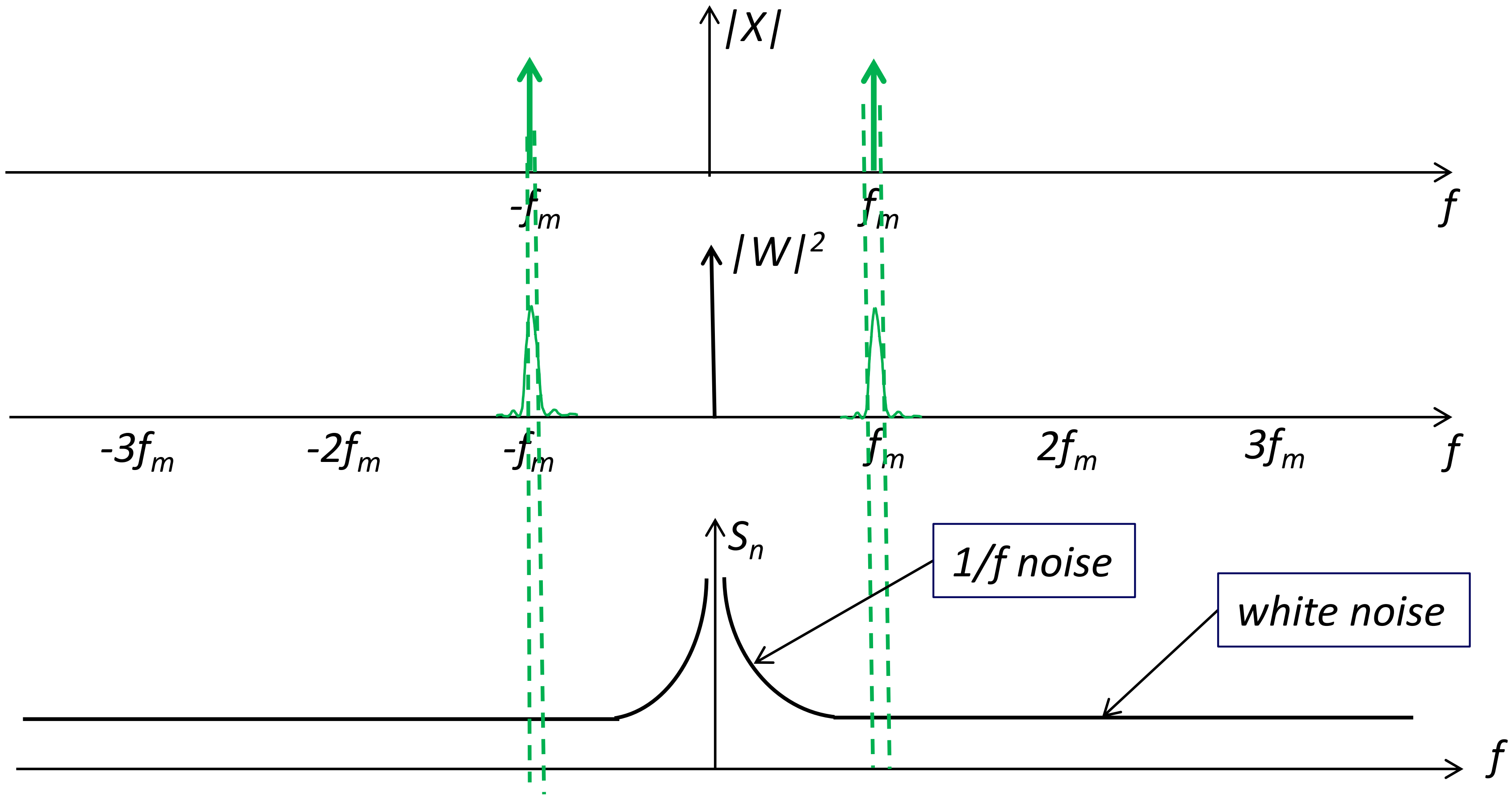
- at f_m **useful** band that collects the **signal** and some white noise around it
- **No more band at f = 0** , no more collection of 1/f noise
- at $3f_m$, $5f_m$ .... residual **harmful** bands that collect just white noise without any signal: how can we get rid also of them?

Continuous sinusoidal weighting instead of peak sampling



TRULY EFFICIENT FILTERING : just one narrow band at f_m

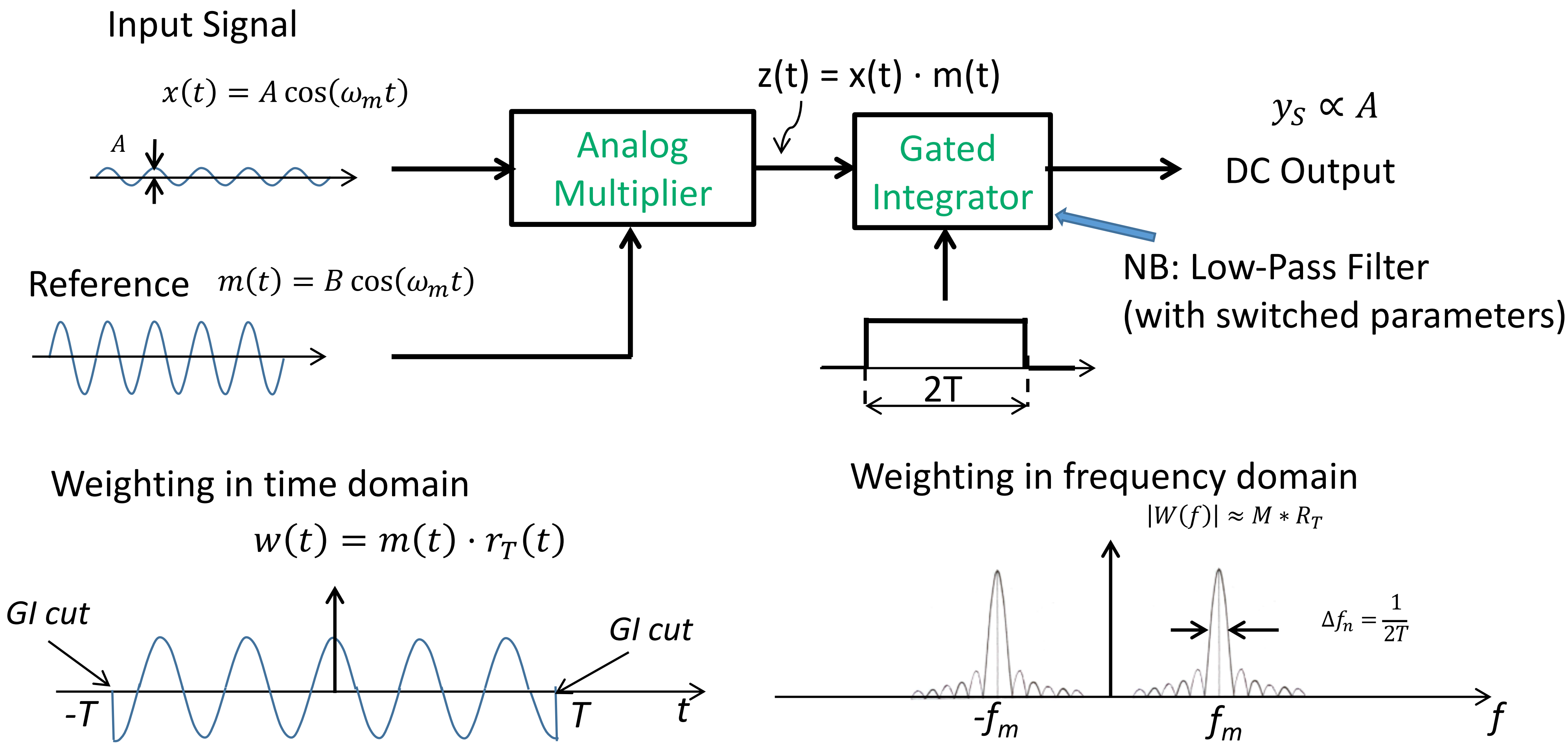
Optimized noise filtering in synchronous measurement



- at f_m **useful** band that collects the **signal** and some white noise around it
- No band at f = 0 , no collection of **1/f noise**
- No residual bands at $3f_m$, $5f_m$ ... no more collection of white noise without any signal

How to implement this optimized synchronous measurement?

Basic set-up for Synchronous Measurement with optimized noise filtering



- **NB** the reference input to the multiplier is a **STANDARD** waveform, which absolutely does **NOT** depend on the signal: it is the same for any signal !!
- Therefore the set-up is a **LINEAR** filter (with time-variant parameters)

Main Advantages of Synchronous Measurements with optimized noise filtering

This linear time-variant filter composed by Analog Multiplier (Demodulator) and Gated Integrator (Low-pass filter with switched-parameter) has a weighting function similar to that of a tuned filter with constant-parameter, but has basic advantages over it:

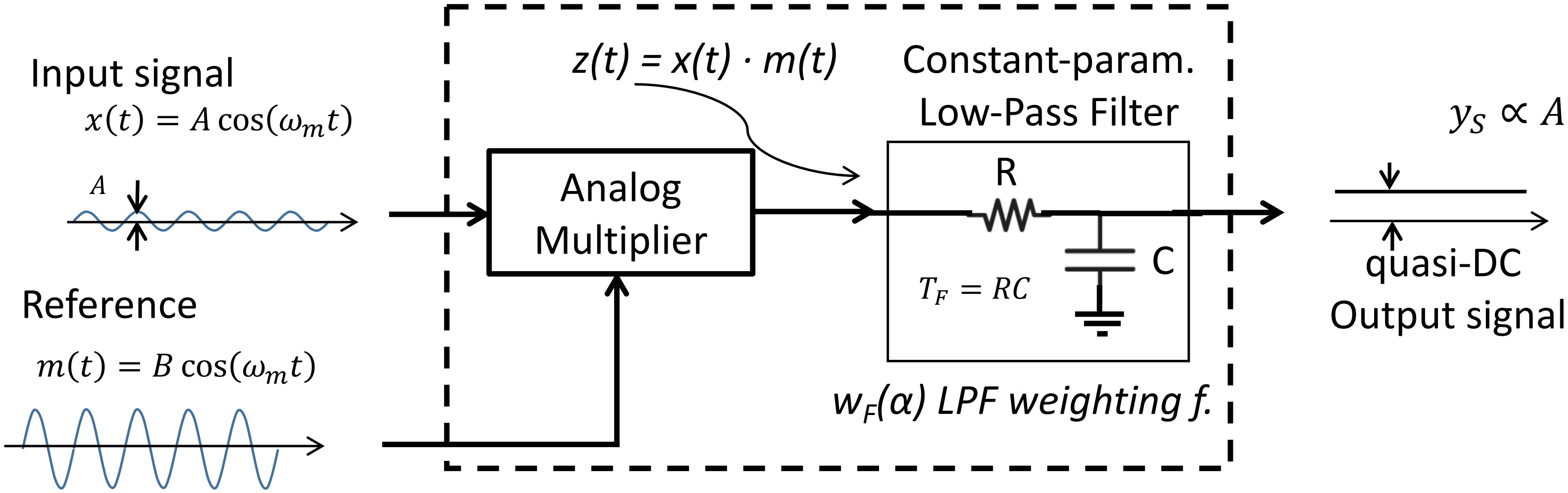
- Center frequency f_m and width Δf_n are **independently** set
- The **center** frequency is set **by the reference** *m(t)* and locked at the frequency f_m
- In cases where f_m has not a very stable value the filter band-center tracks it:
  the signal is thus kept in the admission band even if the width Δf_n is very narrow.
- The **width** $\Delta f_n = 1/2T$ is set **by the GI**, it is the (bilateral) passband of the GI
- Narrow Δf_n and high quality factor Q can thus be easily obtained **at any f_m**

$$\Delta f_n \ll f_m \qquad\qquad Q = \frac{f_m}{\Delta f_n} \gg 1$$

Lock-in Amplifier
Principle and Weighting Function

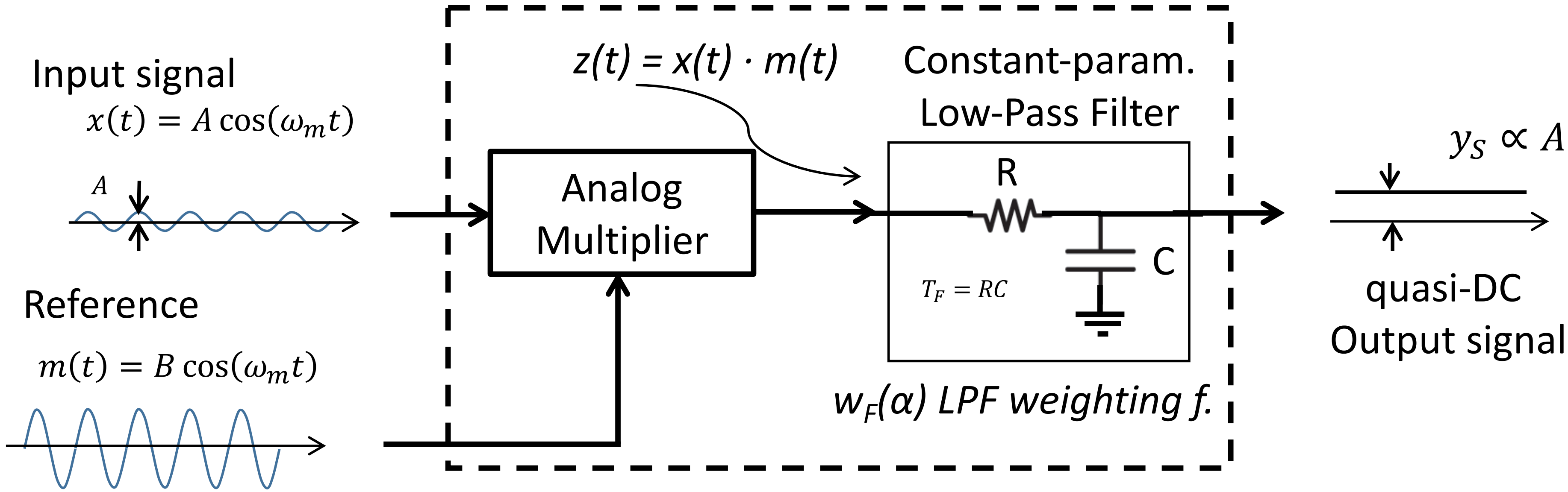
From Discrete to Continuous Synchronous Measurements:
principle of the Lock-in Amplifier (LIA)

With averaging performed by a **gated integrator**, the amplitude A can be measured only at **discrete times** (spaced by at least the averaging time 2T). However, by employing a **constant-parameter low-pass** filter instead of the GI, **continuous monitoring** of the slowly varying amplitude A(t) is obtained.



The **constant** parameter LPF performs a **running** average of the output *z(t)* of the demodulator. The output is continously updated and tracks the slowly varying amplitude *A(t)*. This basic set-up is denoted Phase-Sensitive Detector (PSD) and is the core of the instrument currently called **Lock-in Amplifier**.

Principle of the Lock-in Amplifier (LIA)



The **constant** parameter LPF performs a **running** average of *z(t)* over a few T_F that continously updates the output

$$y(t) = \int_0^\infty z(\alpha)\, w_F(\alpha) d\alpha = \int_0^\infty x(\alpha)\, m(\alpha) w_F(\alpha) d\alpha$$

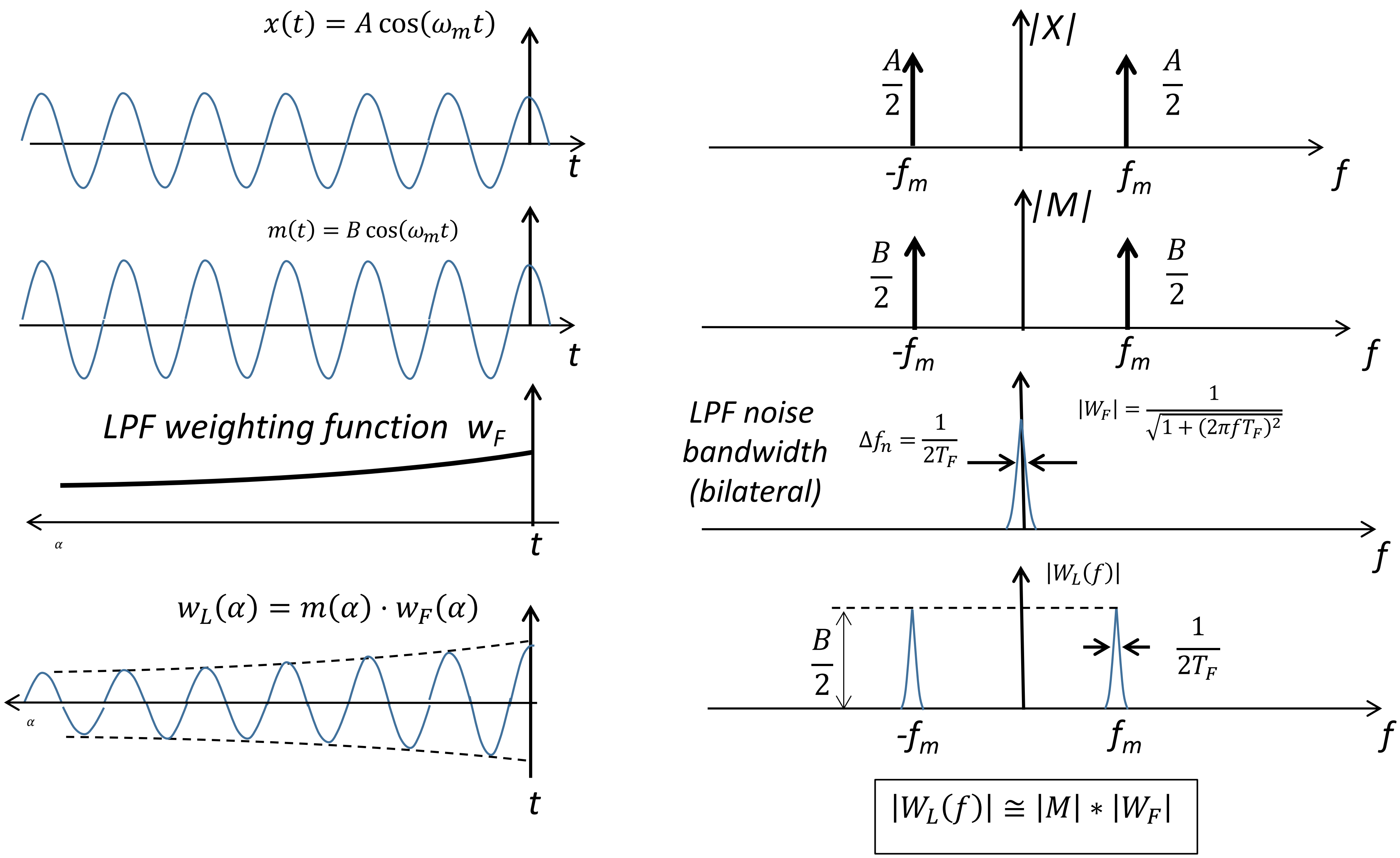
By comparison with the definition of the LIA weighting function $w_L(\alpha)$

$$y(t) = \int_0^\infty x(\alpha)\, w_L(\alpha) d\alpha$$

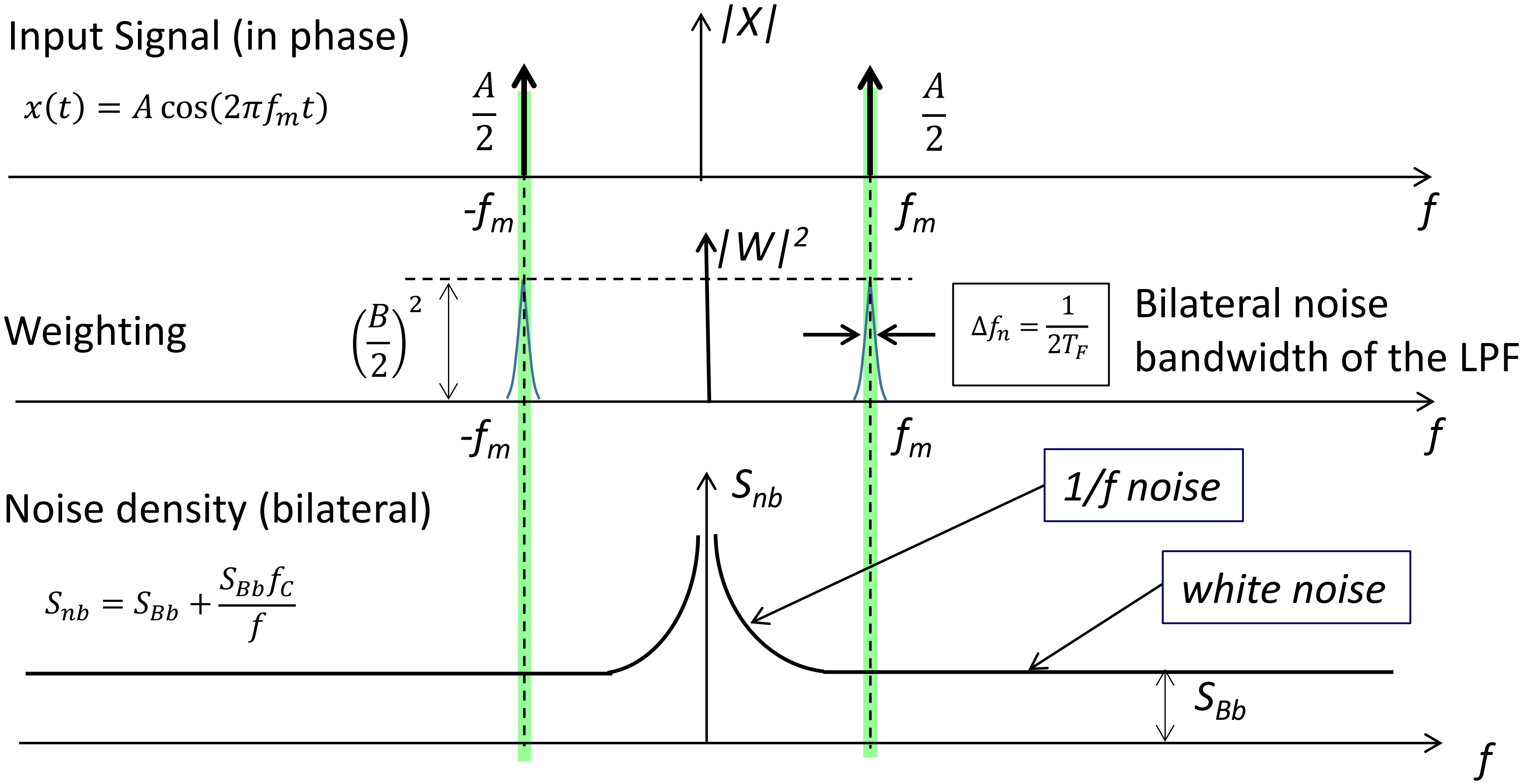
we see how the **demodulation** and LPF are combined in the LIA

$$w_L(\alpha) = m(\alpha) \cdot w_F(\alpha) \quad \rightleftarrows \quad |W_L(f)| \cong |M| * |W_F|$$

Weighting Function w_L of the Lock-in Amplifier



S/N of the Lock-in Amplifier



Output signal $y_S = 2\frac{A}{2}\cdot\frac{B}{2} = \frac{B}{2}A$

Output Noise $\overline{n_{yL}^2} = 2\left(\frac{B}{2}\right)^2 \cdot S_{Bb}\cdot\Delta f_n = \frac{B^2}{2}\cdot S_{Bb}\cdot\Delta f_n$

$$\left(\frac{S}{N}\right)_L = \frac{y_S}{\sqrt{\overline{n_{yL}^2}}} = \frac{A}{\sqrt{2 S_{Bb}\Delta f_n}}$$

S/N of the Lock-in Amplifier

$$\left(\frac{S}{N}\right)_L = \frac{A}{\sqrt{2S_{Bb}\Delta f_n}}$$

or in power terms

$$\left(\frac{S}{N}\right)_L^2 = \frac{\frac{A^2}{2}}{S_{Bb}\Delta f_n} = \frac{in - phase\ signal\ power}{half power\ of white\ noise\ in\ the\ band\ \Delta f_n}$$

S/N equation in terms of the unilateral parameters

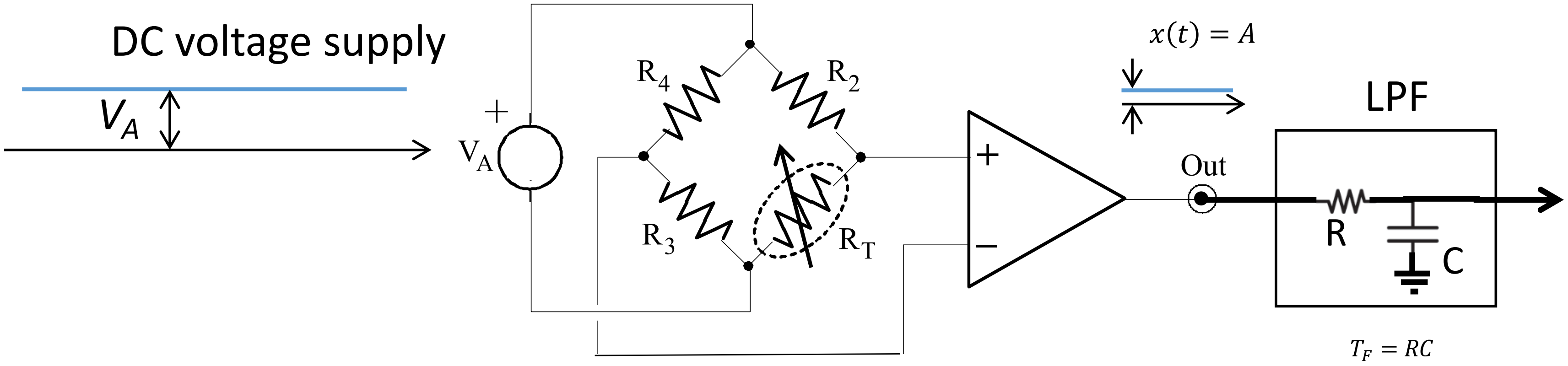
By introducing

- f_{Fn} the LPF unilateral bandwidth (upper band-limit for noise), i.e. $\Delta f_n = 2f_{Fn}$
- S_{bu} the unilateral noise density, i.e. $2S_{Bb} = S_{Bu}$

we can write

$$\left(\frac{S}{N}\right)_L = \frac{A}{\sqrt{2S_{Bu}f_{Fn}}}$$

Case of DC signal with LPF compared to AC signal with LIA



Let us consider the set-up of the key example (measurement with resistive sensor) now with DC supply voltage V_A equal to the amplitude of the previous AC supply. The signal now is a DC voltage equal to the amplitude A of the previous AC signal.

With a LPF equal to that employed in the previous LIA we obtain:

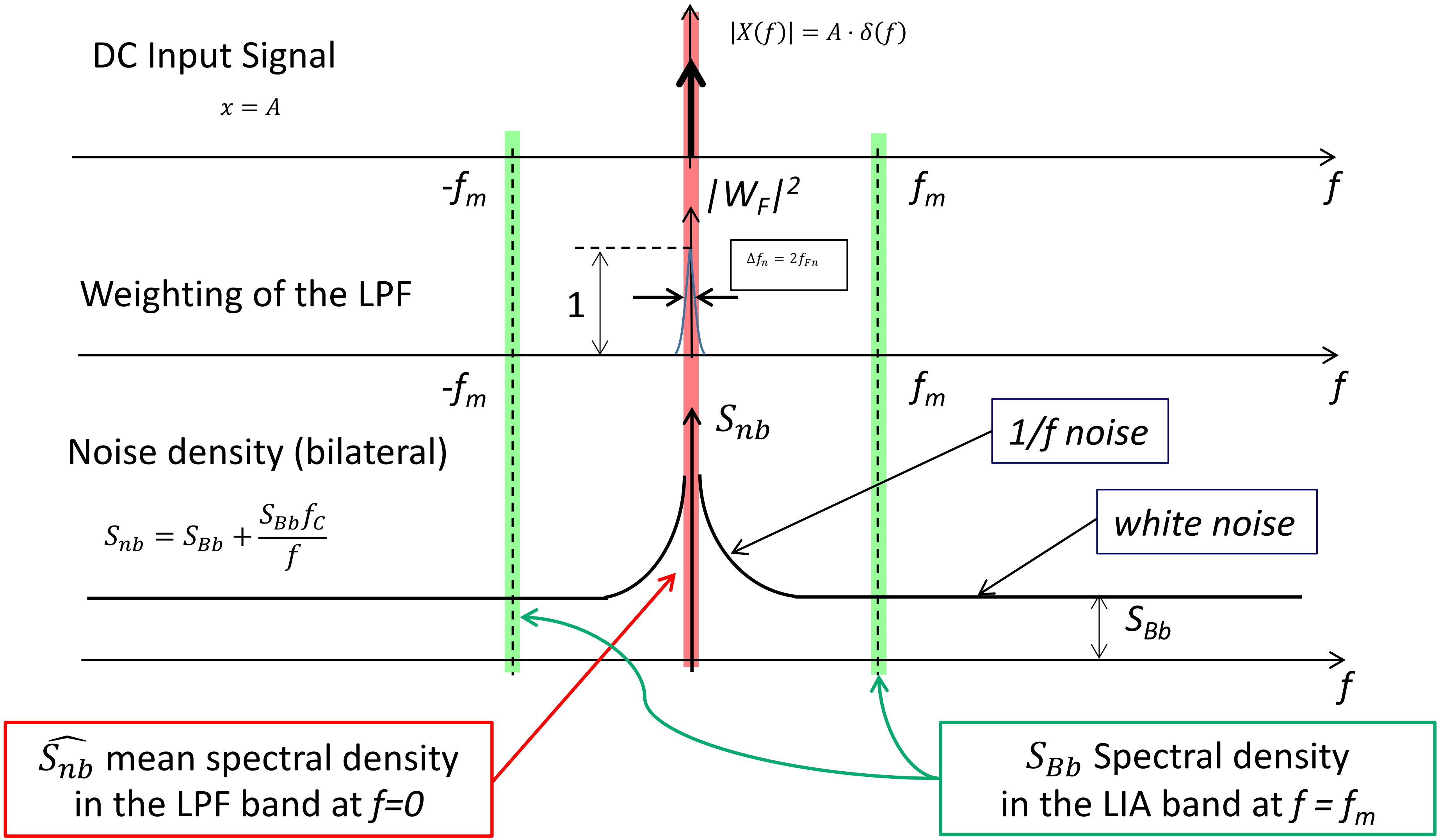
- Output signal $y_C = A$
- Output Noise $\overline{n_{yC}^2} = \widehat{S_{nu}} \cdot f_{Fn}$
  ($\widehat{S_{nu}}$ mean density in the LPF band)

$$\left(\frac{S}{N}\right)_C = \frac{y_C}{\sqrt{\overline{n_{yC}^2}}} = \frac{A}{\sqrt{\widehat{S_{nu}} f_{Fn}}}$$

This S/N may look **better by the factor $\sqrt{2}$** than the S/N obtained with the LIA, but is this conclusion true?

NO, such a conclusion is **grossly wrong because $\widehat{S_{nu}} \gg S_{Bu}$!!**

DC signal and LPF compared to AC signal and LIA



A passband at $f = 0$ is a risk: 1/f noise gives $\widehat{S_{nb}} \gg S_{Bb}$!!

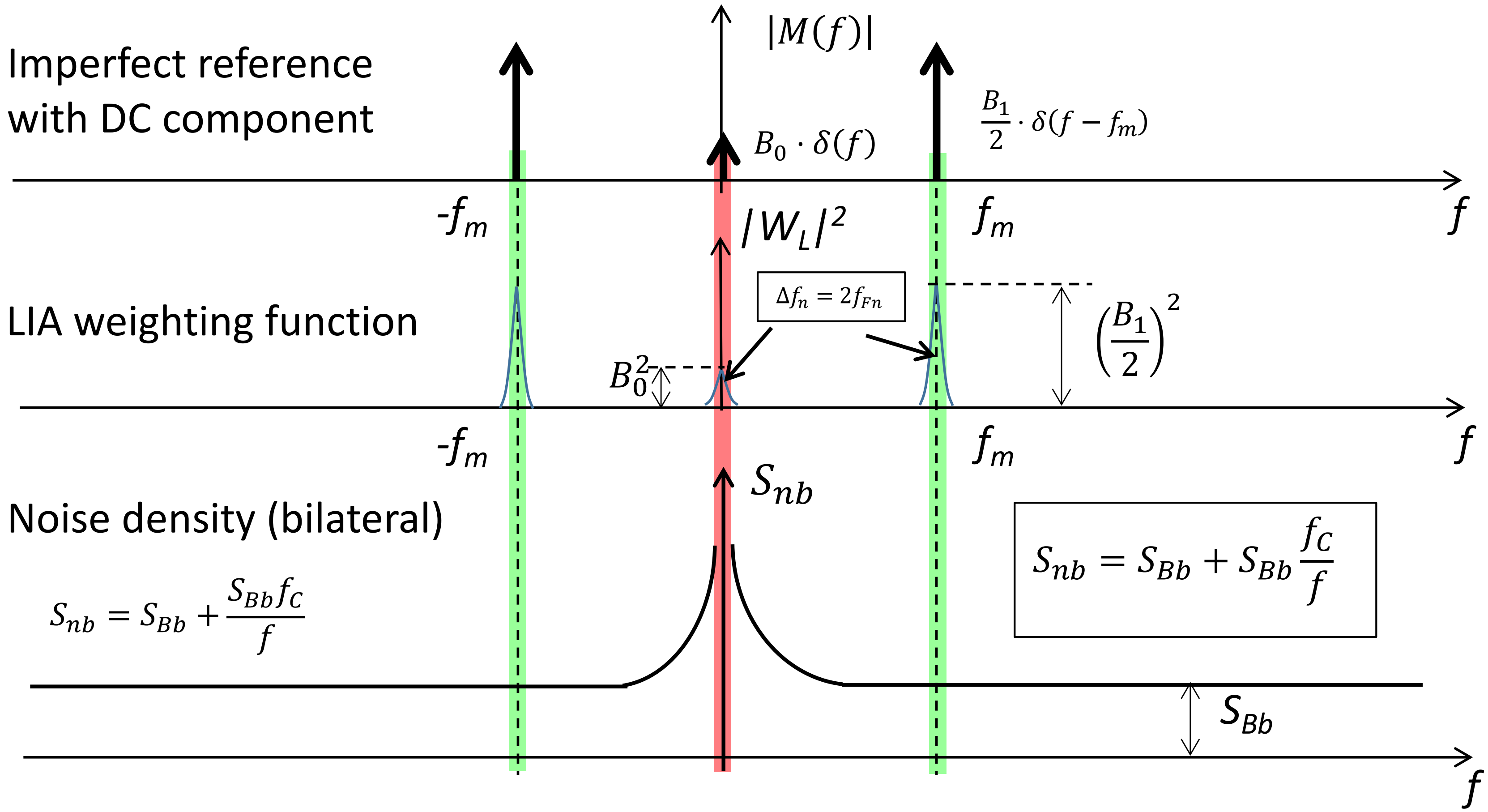
Fake LIA passbands arise from imperfect modulation

- Ideally, the reference waveform should be a perfect sinusoid at frequency f_m with amplitude B_1
- In reality, deviations from the ideal can generate spurious harmonics at multiples kf_m $(k = 0, 1, 2 ...)$ with amplitudes B_k (small $B_k << B_1$ in case of small deviations)
- Moreover, effects equivalent to an imperfect reference waveform can be caused by non-ideal operation (non-linearity) of the multiplier
- Since it is

$$|W_L(f)| \cong M(f) * W_F(f)$$

  each spurious harmonic component of $M(f)$ adds to the LIA weighting function W_L a spurious passband at frequency kf_m with amplitude B_k and shape given by the LPF
- A fake passband **at $f = 0$ is particularly detrimental even with small $B_0 << B_1$** because it covers the high spectral density of $1/f$ noise ....
- .... and unluckily any **deviation from perfect balance of positive and negative areas** of the reference produces a DC component with associated passband at $f = 0$!!

Fake LIA passband at f = 0



The ratio $\widehat{S_{nu}}/S_{Bu} > f_C/f_{Fn} \gg 1$ can match or exceed the amplitude ratio $\frac{B_1^2}{2B_0^2}$

so that the noise in the fake passband $\overline{n_{yL0}^2} = B_0^2 \cdot \widehat{S_{nb}} \cdot \Delta f_n = B_0^2 \cdot \widehat{S_{nu}} \cdot f_{Fn}$

can equal or exceed that in the correct passband $\overline{n_{yL1}^2} = \frac{B_1^2}{2} \cdot S_{Bb} \cdot \Delta f_n = \frac{B_1^2}{2} \cdot S_{Bu} \cdot f_{Fn}$