COURSE OUTLINE

- Introduction
- Signals and Noise
- Filtering: Band-Pass Filters 1 - BPF1
- Sensors and associated electronics

Band-Pass Filters 1

- Narrow-Band Signals
- Recovering Narrow-Band Signals from Noise
- Moving Signals in Frequency (Signal Modulation)

Narrow-Band Signals

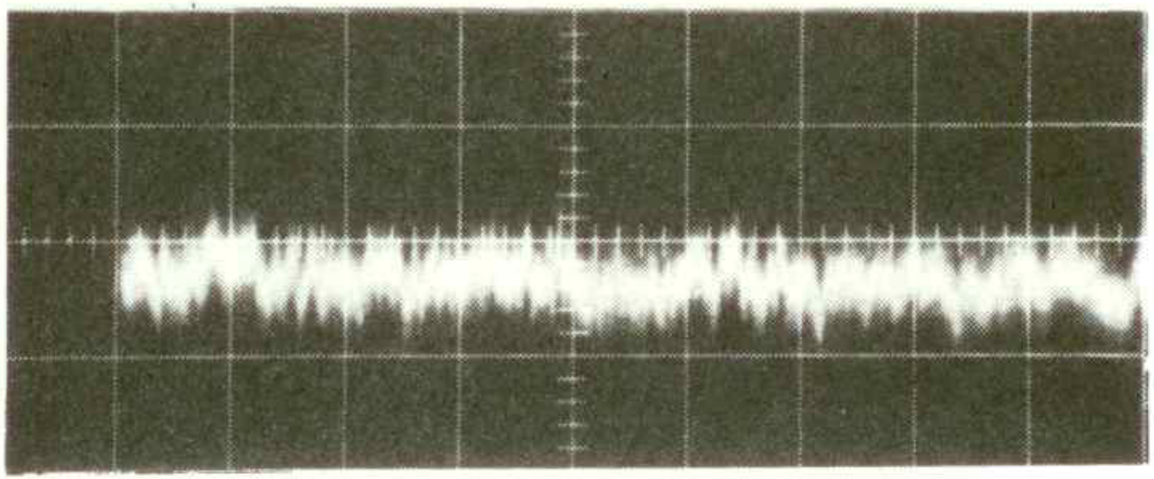
Narrow-Band Signals

Power signals with a narrow power spectrum, that is, a peak with

- center-frequency f_s
- bandwidth Δf_s which is small in absolute value, typically $\Delta f_s < 10$ Hz, and/or with respect to the center frequency $\Delta f_s \ll f_s$

They approximate well a sinusoid over a wide time interval $T_s \approx 1/\Delta f_s$

QUESTION: how can we measure such narrow-band signals in presence of intense white noise? And what if also 1/f noise is present?



Recovering Narrow-Band Signals from Noise

Recovering Narrow-Band Signals from noise

Let's see some typical examples of signals with

- narrow linewidth $\Delta f_s = 1\ Hz$
- small amplitude $V_s \leq 100\ nV$

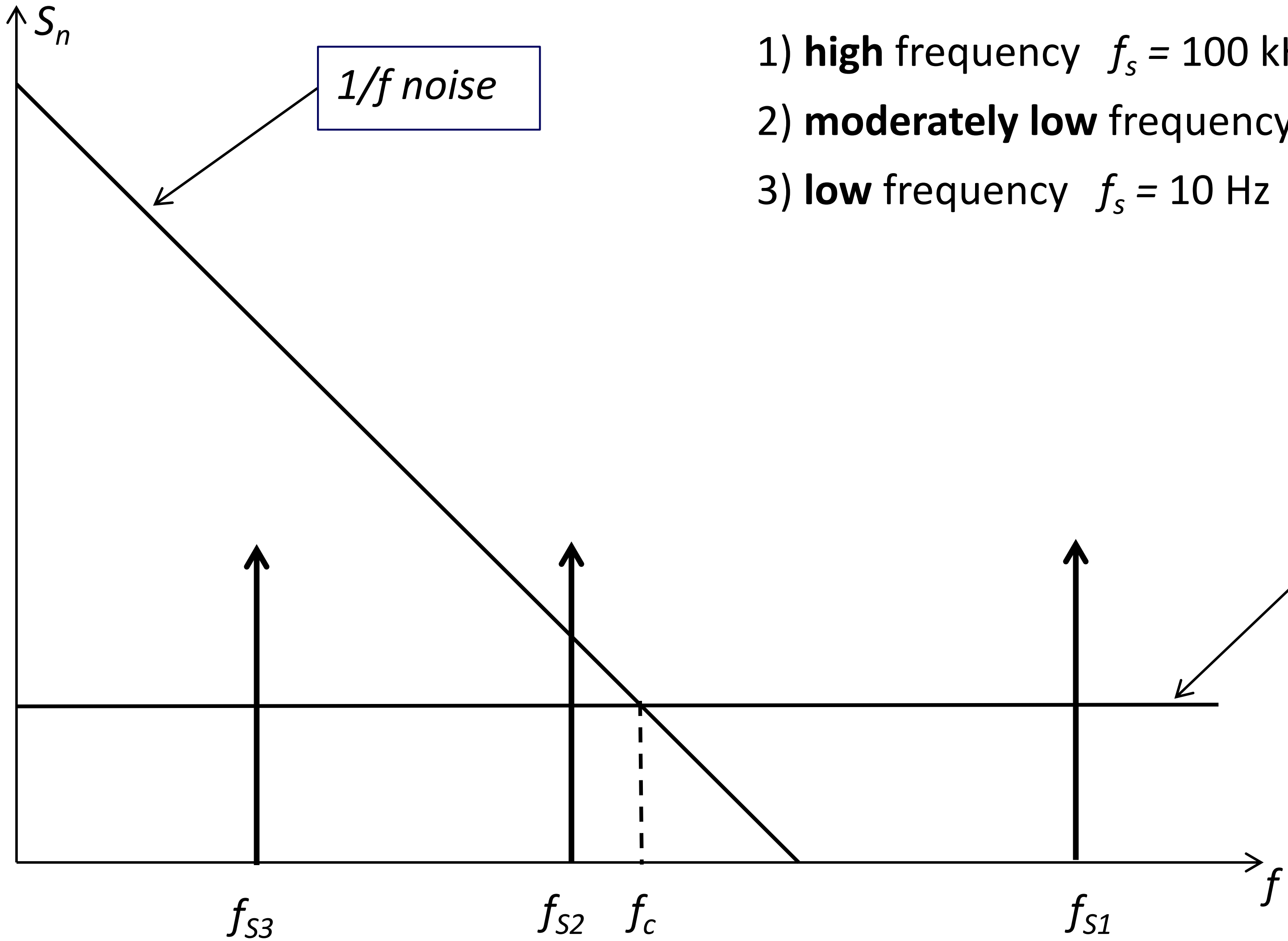
for bringing them to higher level (suitable for processing circuits: filters, meters, etc.) they are amplified by a DC-coupled wide-band preamplifier with

- upper band-limit $f_h = 1MHz$
- noise spectral density (referred to input) with
  «white» component $\sqrt{S_b} = 5nV/\sqrt{Hz}$
  and *1/f* component with corner frequency $f_c = 2kHz$

Let us consider **three cases** with different center-frequency f_s :

- Case 1: **high** frequency f_s = 100 kHz
- Case 2: **moderately low** frequency f_s = 1 kHz
- Case 3: **low** frequency f_s = 10 Hz

Recovering Narrow-Band Signals



Three cases with different center-frequency f_s :

1) **high** frequency f_s = 100 kHz
2) **moderately low** frequency f_s = 1 kHz
3) **low** frequency f_s = 10 Hz

CASE 1: signal $V_s \leq 100\ nV$ at high frequency $f_s = 100$ kHz

a) observing the voltage waveform in the TIME DOMAIN, i.e. on oscilloscope display

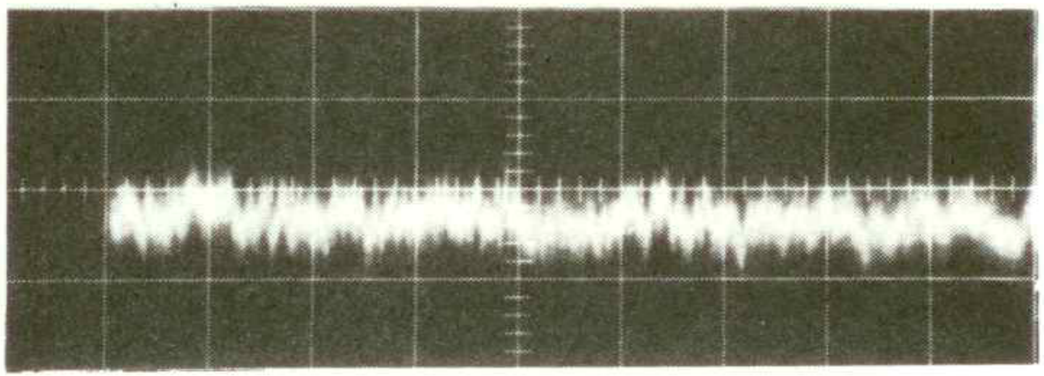
The signal to be recovered is at frequency f_s = 100 kHz much higher than the noise corner frequency $f_c = 2kHz$, so that we can use a simple high-pass filter with band-limit f_i = 10kHz to cut off the 1/f noise and obtain a rms noise (referred to the preamp input)

$$\sqrt{\overline{v_n^2}} = \sqrt{S_b} \cdot \sqrt{(f_h - f_i)} \approx \sqrt{S_b} \cdot \sqrt{f_h} = 5\mu V$$

and therefore

$$\frac{S}{N} = \frac{V_S}{\sqrt{\overline{v_n^2}}} \leq 0{,}02 << 1$$

Even the highest signal $V_s = 100\ nV$ is **practically invisible on the oscilloscope display**! The noise covers a band $\approx 5 \times rms\ value \approx 20\ \mu V$ and the sinusoidal signal is buried in it!



Vertical scale 50µV/div

Horizontal scale 5µs/div

Recovering Narrow-Band Signals

CASE 1: signal $V_s \leq 100\ nV$ at high frequency f_s = 100 kHz

b) observing the power spectrum in FREQUENCY DOMAIN, i.e. on spectrum analyzer display

SIGNAL: the power $P_S = \frac{V_S^2}{2} = 50 \cdot 10^{-16}\ V^2$ is within a bandwidth $\Delta f_S = 1\ Hz$

so that the effective power density of the signal is $\sqrt{S_S} \approx \sqrt{\frac{P_S}{\Delta f_S}} = \frac{70nV}{\sqrt{Hz}}$

NOISE: the effective power density at f_s = 100 kHz is $\sqrt{S_b} = 5nV/\sqrt{Hz}$

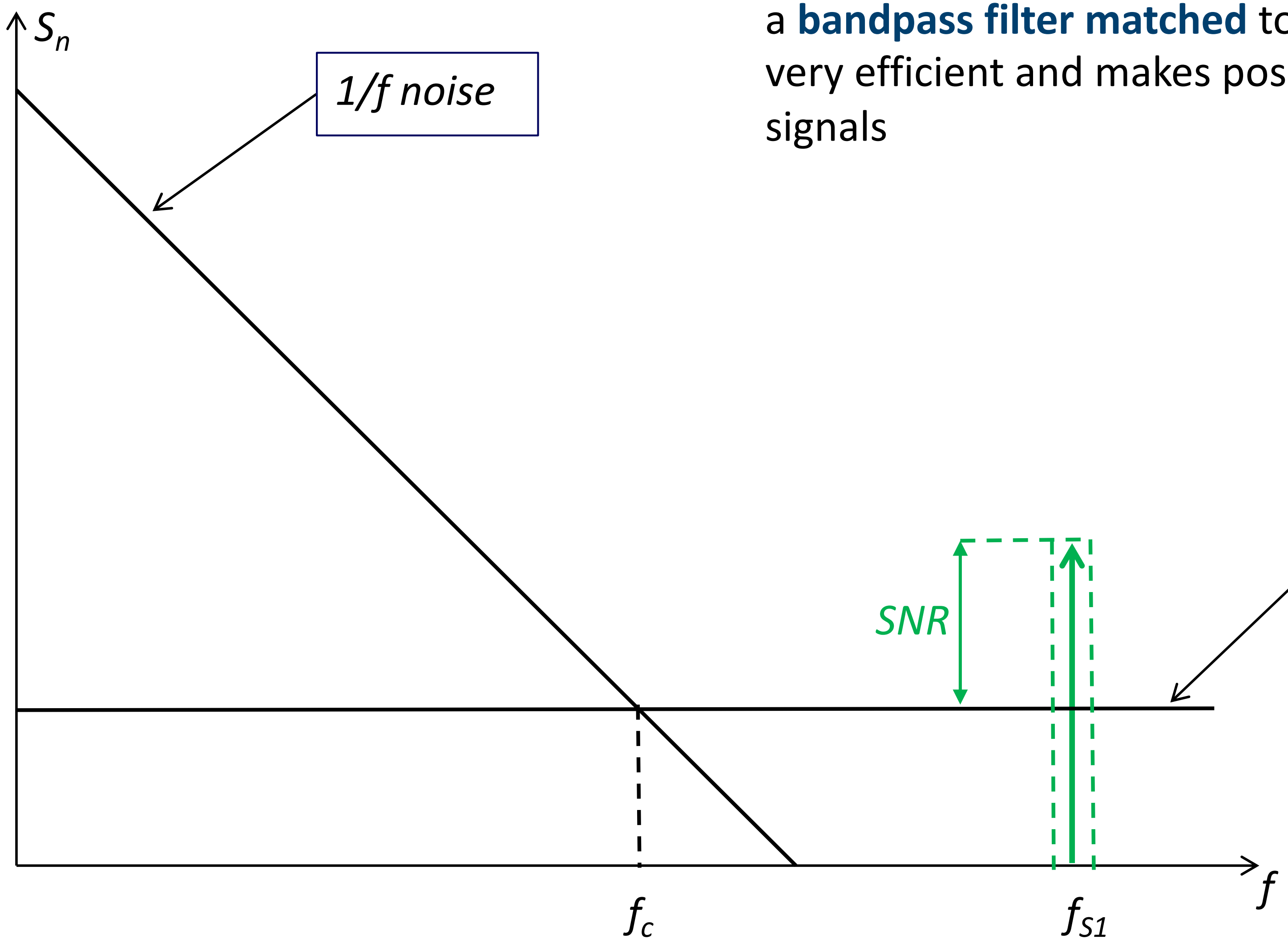
On the spectrum analyzer display the signal peak is **very well visible above the noise!**

$$\frac{\sqrt{S_S}}{\sqrt{S_b}} = 14 \gg 1$$

Conclusion: good S/N can be obtained with a bandpass filter having bandwidth Δf_b matched to the signal band $\Delta f_b \approx \Delta f_s$

$$\frac{S}{N} = \sqrt{\frac{P_S}{S_b \Delta f_b}} = \sqrt{\frac{S_S \Delta f_S}{S_b \Delta f_b}} \approx \frac{\sqrt{S_S}}{\sqrt{S_b}} = 14 \gg 1$$

Recovering Narrow-Band Signals



For a **narrow-band signal** buried in white noise a **bandpass filter matched** to the signal band is very efficient and makes possible to recover signals

Recovering Narrow-Band Signals

CASE 2: signal $V_s \leq 100\ nV$ at moderately low frequency $f_s = 1$ kHz

a) observing the voltage waveform in the TIME DOMAIN, i.e. on oscilloscope display

The signal is now at $f_s = 1$ kHz just below the corner frequency $f_c = 2kHz$.
For reducing the 1/f noise we can still use a high-pass filter, but in order to pass the signal the band-limit f_i must be reduced: $f_i << f_s = 1$ kHz, typically $f_i = 100$Hz.
The rms noise referred to the input is

$$\sqrt{\overline{v_n^2}} \approx \sqrt{S_b(f_h - f_i) + S_b f_c \ln\left(\frac{f_h}{f_i}\right)} \approx \sqrt{S_b f_h + S_b f_c \ln\left(\frac{f_h}{f_i}\right)} \approx \sqrt{S_b f_h} \approx 5\mu V$$

1/f noise is negligible $S_b f_c \ln\left(\frac{f_h}{f_i}\right) \ll S_b f_h$

and therefore

$$\frac{S}{N} = \frac{V_S}{\sqrt{\overline{v_n^2}}} \leq 0{,}02 << 1$$

The situation is practically equal to that of Case 1: the signal is **practically invisible on the oscilloscope display, it's buried in the noise**!

Recovering Narrow-Band Signals

CASE 2: signal $V_s \leq 100\ nV$ at moderately low frequency f_s = 1 kHz

b) observing the power spectrum in FREQUENCY DOMAIN, i.e. on spectrum analyzer display

SIGNAL: the power $P_S = \frac{V_S^2}{2} = 50 \cdot 10^{-16}\ V^2$ is within a bandwidth $\Delta f_S = 1\ Hz$

so that the effective power density of the signal is $\sqrt{S_S} \approx \sqrt{\frac{P_S}{\Delta f_S}} = \frac{70nV}{\sqrt{Hz}}$

NOISE: due to the **1/f noise**, the effective power density at f_s = 1 kHz is somewhat higher

$$\sqrt{S_n(f_s)} = \sqrt{S_b + S_b \frac{f_c}{f_s}} = \sqrt{S_b}\sqrt{1 + \frac{f_c}{f_s}} = \sqrt{3} \cdot \sqrt{S_b} \approx 8{,}7 \frac{nV}{\sqrt{Hz}}$$

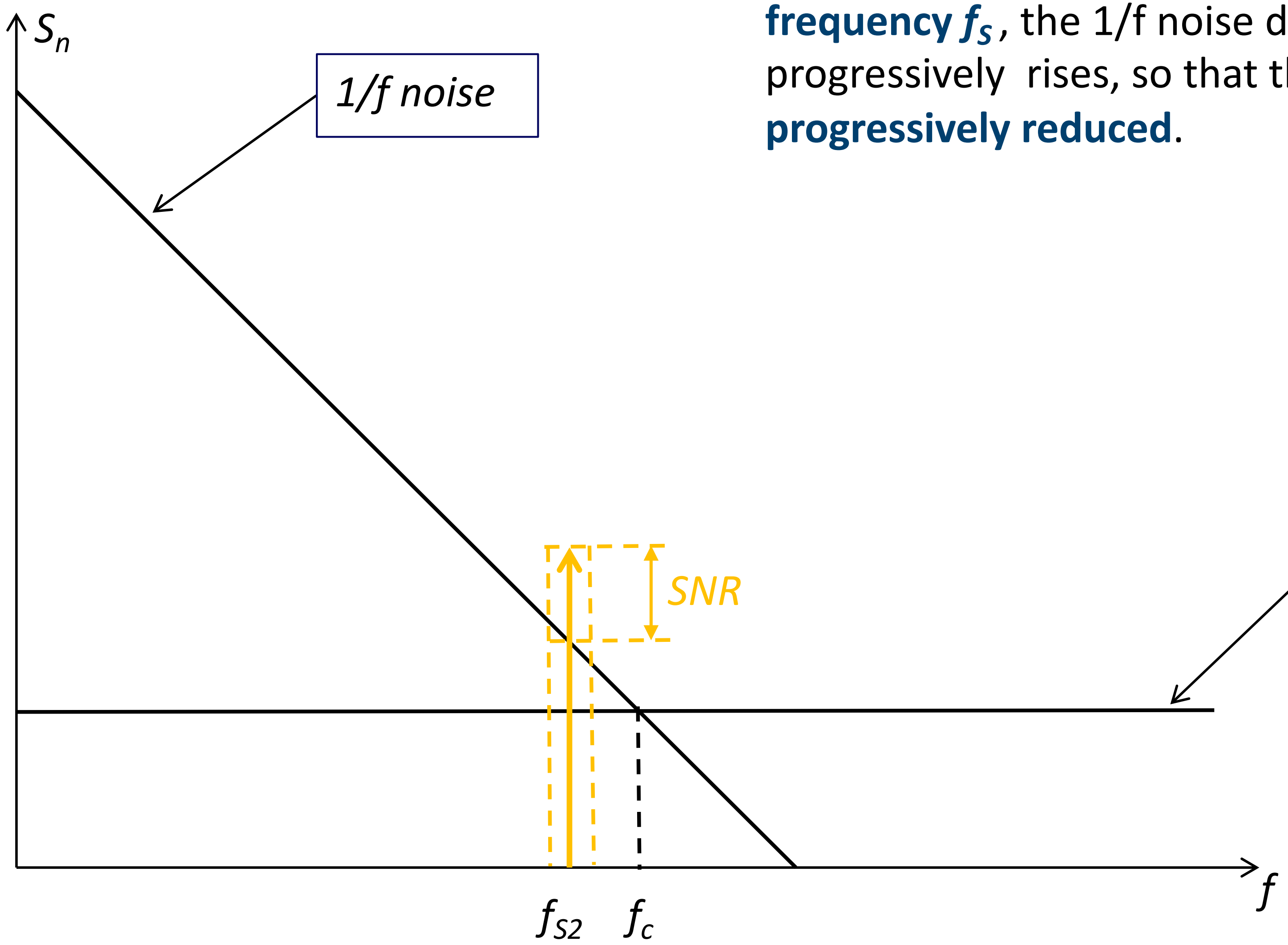
Anyway, on the spectrum analyzer display the signal peak is still **well visible above the noise**

$$\frac{\sqrt{S_S}}{\sqrt{S_n(f_S)}} = 8 > 1$$

Conclusion: a bandpass filter with bandwidth Δf_b matched to the signal $\Delta f_b \approx \Delta f_s$ still gives a **fairly good S/N**

$$\frac{S}{N} = \sqrt{\frac{P_S}{S_n(f_S)\Delta f_b}} = \sqrt{\frac{S_S \Delta f_S}{S_n(f_S)\Delta f_b}} \approx \frac{\sqrt{S_S}}{\sqrt{S_n(f_S)}} = 8 > 1$$

Recovering Narrow-Band Signals



If we consider signals at **progressively lower frequency f_S**, the 1/f noise density at f_S progressively rises, so that the available **S/N is progressively reduced**.

Recovering Narrow-Band Signals

CASE 3: signal $V_s \le 100\ nV$ at low frequency f_s = 10 Hz

a) observing the voltage waveform in the TIME DOMAIN, i.e. on oscilloscope display

The signal is now at f_s = 10 Hz much below the corner frequency f_c = *2kHz.*
For reducing the the 1/f noise we can still use a high-pass filter, but with strongly reduced band-limit $f_i << f_s$ = 10 Hz, typically f_i = 1 Hz. The rms noise referred to input is

$$\sqrt{\overline{v_n^2}} \approx \sqrt{S_b(f_h - f_i) + S_b f_c \ln\left(\frac{f_h}{f_i}\right)} \approx \sqrt{S_b f_h + S_b f_c \ln\left(\frac{f_h}{f_i}\right)} \approx \sqrt{S_b f_h} \approx 5\mu V$$

1/f noise is negligible $S_b f_c \ln\left(\frac{f_h}{f_i}\right) \ll S_b f_h$

and therefore $$\frac{S}{N} = \frac{V_S}{\sqrt{\overline{v_n^2}}} \le 0{,}02 << 1$$

The situation is practically equal to that of Case 1 : the signal is **practically invisible on the oscilloscope display, it's buried in the noise**!

Recovering Narrow-Band Signals

CASE 3: signal $V_s \leq 100\ nV$ at low frequency f_s = 10 Hz

b) observing the power spectrum in FREQUENCY DOMAIN, i.e. on spectrum analyzer display

SIGNAL: the power $P_S = \frac{V_S^2}{2} = 50 \cdot 10^{-16}\ V^2$ is within a bandwidth $\Delta f_S = 1\ Hz$

so that the effective power density of the signal is $\sqrt{S_S} \approx \sqrt{\frac{P_S}{\Delta f_S}} = 70 \frac{nV}{\sqrt{Hz}}$

NOISE: due to the **1/f noise**, the effective power density at f_s = 10 Hz is now **much higher**

$$\sqrt{S_n(f_s)} = \sqrt{S_b + S_b \frac{f_c}{f_s}} = \sqrt{S_b}\sqrt{1 + \frac{f_c}{f_s}} = 14{,}2 \cdot \sqrt{S_b} \approx 71 \frac{nV}{\sqrt{Hz}}$$

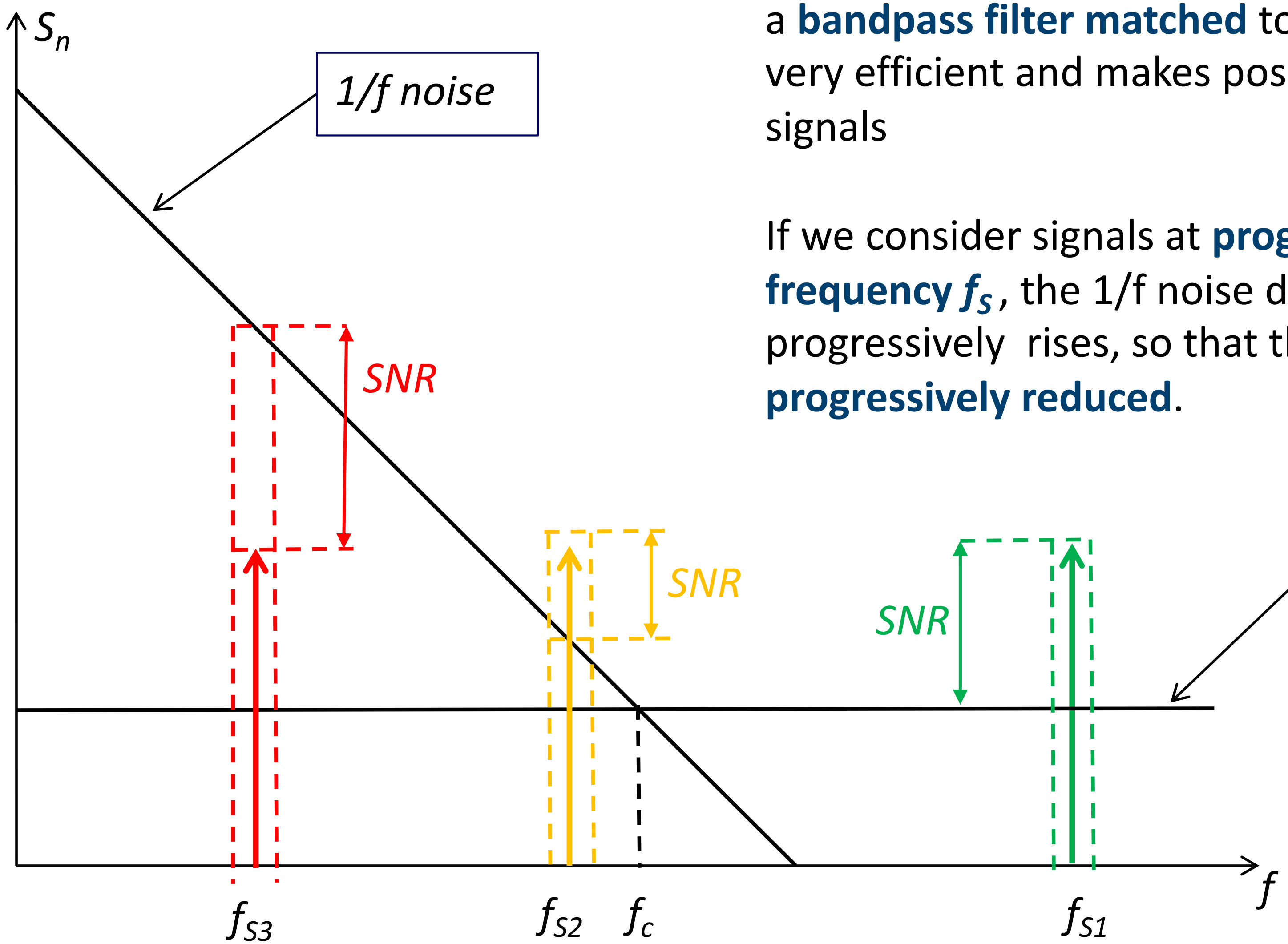
On the spectrum analyzer display the signal peak is **barely visible, it's equal to the noise**

$$\frac{\sqrt{S_S}}{\sqrt{S_n(f_S)}} \approx 1$$

Conclusion: **the S/N is insufficient** even with a bandpass filter with narrow bandwidth Δf_b matched to the signal $\Delta f_b \approx \Delta f_s$

$$\frac{S}{N} = \sqrt{\frac{P_S}{S_b \Delta f_b}} = \sqrt{\frac{S_S \Delta f_S}{S_n(f_S)\Delta f_b}} \approx \frac{\sqrt{S_S}}{\sqrt{S_n(f_S)}} \leq 1$$

Summary



For a **narrow-band signal** buried in white noise a **bandpass filter matched** to the signal band is very efficient and makes possible to recover signals

If we consider signals at **progressively lower frequency f_S**, the 1/f noise density at f_S progressively rises, so that the available **S/N is progressively reduced**.

OPEN QUESTIONS

- We need **efficient band-pass filters** with very narrow band-width.

  but we shall deal with this task after dealing with the following question.

- If the information is carried by the amplitude of a low-frequency signal, it has to face also 1/f noise. ***It would be advantageous to escape this noise by preliminarily transferring the information to a signal at higher frequency***.

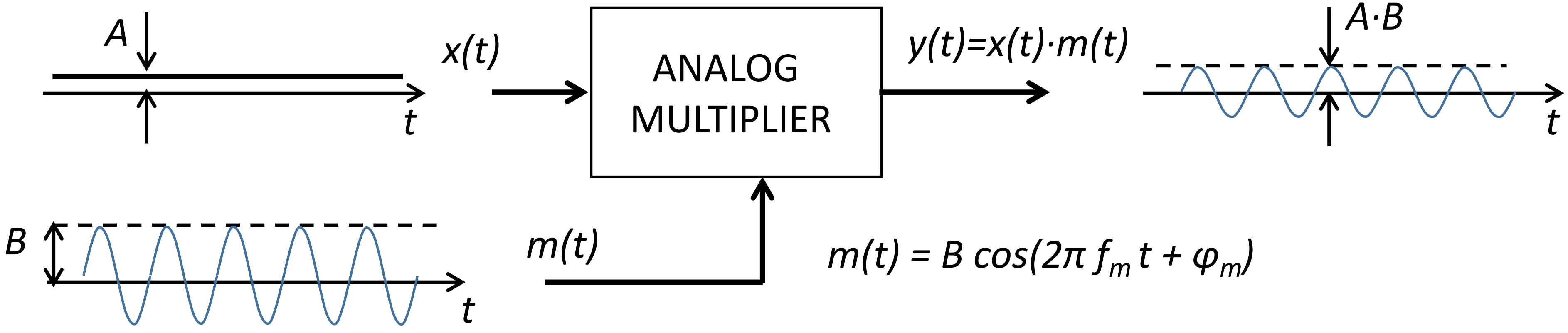
- However: **how can we transfer the signal to higher frequency**?

- For escaping 1/f noise, a low-frequency signal should be transferred to higher frequency **before it mixes with 1/f noise**

- **The frequency-transfer stages have their own noise!**

Moving Signals in Frequency
(Signal Modulation)

Amplitude Modulation with DC signal (ideal)

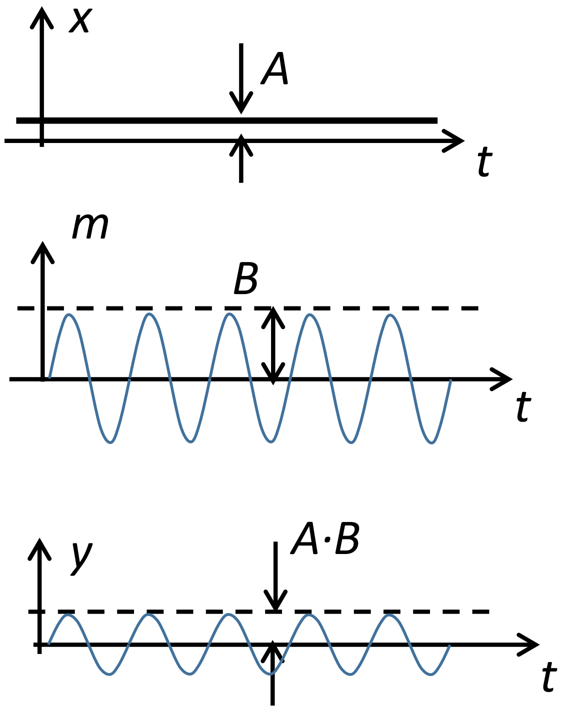


- **Information is brought by** the (VARIABLE) amplitude A of a DC signal $x(t) = A$.
  (NB: a real DC signal is a signal at very low frequency with very narrow bandwidth)
- An analog multiplier circuit combines the signal with a sinusoidal waveform $m(t)$ *(called reference or carrier)* with frequency f_m and CONSTANT amplitude B
- The information is transferred to the amplitude of a sinusoidal signal $y(t)$ at frequency f_m

$$y(t) = A \cdot B \cos(2\pi f_m t + \varphi_m)$$

Amplitude Modulation with DC signal (ideal)

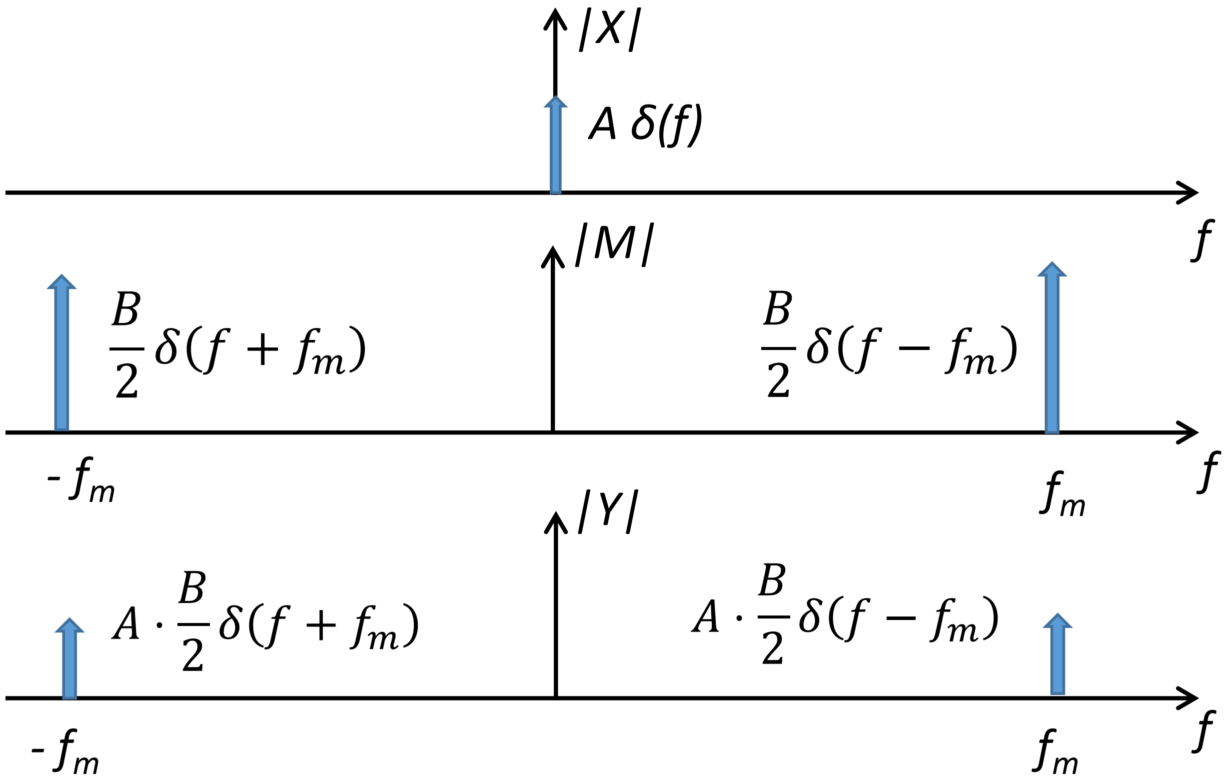
TIME DOMAIN



$y(t) = x(t) \cdot m(t) =$

$= A \cdot B \cos(2\pi f_m t + \varphi_m)$

FREQUENCY DOMAIN



$Y(f) = X(f) * M(f)$

The signal is shifted in frequency by $+f_m$ and $-f_m$ and in phase by $+\varphi_m$ and $-\varphi_m$ respectively

Convolution in the Frequency Domain

In the **time domain (TD)** the amplitude modulation is the **multiplication** of the signal *x(t)* (with variable amplitude A) by the reference waveform *m(t)* (with standard amplitude B)

$$y(t) = x(t) \cdot m(t)$$

In the **frequency domain (FD)** it is the **convolution** of the transformed signal *X(f)* by the transformed reference M*(f)*

$$Y(f) = X(f) * M(f) = \int_{-\infty}^{\infty} X(\alpha)\, M(f-\alpha) d\alpha$$

Convolution is more complicated in FD than in TD because:

1. the functions to be convolved are twofold, that is, they run in the positive and negative sense of the frequency axis
2. **Complex** values must be summed at every frequency for obtaining *Y(f)*.

In general the result of FD convolution is not as intuitive as that of TD convolution and the module |Y(f)|is **NOT** given by the convolution of |X(f)| and|M(f)|

$$|Y(f)| \neq |X(f)| * |M(f)|$$

we must first compute the real and imaginary parts of Y(f) and then obtain |Y(f)|

Amplitude Modulation with Narrow-Band Signal

In the cases here considered, however, the issue is remarkably simplified because

a) *X(f)* is confined in a **narrow** bandwidth Δf_s

b) *M(f)* has a line spectrum with (fundamental) frequency f_m that is much greater than the signal bandwidth $\boldsymbol{f_m >> \Delta f_s}$

In the convolution *X(f) * M(f)* each line of M(f) acts on *X(f)* as follows

- Shifts in frequency every component of *X(f)* by + $\boldsymbol{f_m}$ and $\boldsymbol{-f_m}$ (i.e. adds to each frequency + $\boldsymbol{f_m}$ and $\boldsymbol{-f_m}$)
- Shifts in phase every component of *X(f)* by $+\varphi_m$ and $-\varphi_m$ (i.e. adds to every phase $+\varphi_m$ and $-\varphi_m$)

*In cases with $\Delta f_s << \boldsymbol{f_m}$, there is **no sum of complex numbers** to be computed because at any frequency f there is at most one term to be considered, all other terms are negligible.*

The result of the convolution is easily visualized: every line of *M(f)* shifts *X(f)* in frequency and adds to *X(f)* its phase. Therefore, |Y(f)| is well approximated by the convolution of |X(f)| and |M(f)| and $|Y(f)|^2$ by the convolution of $|X(f)|^2$ and $|M(f)|^2$

$$\boxed{|Y(f)| \cong |X(f)| * |M(f)|}$$

$$\boxed{|Y(f)|^2 \cong |X(f)|^2 * |M(f)|^2}$$

Amplitude Modulation with Narrow-Band Signal

Example of quasi-DC

NB signal (with very long T_S)

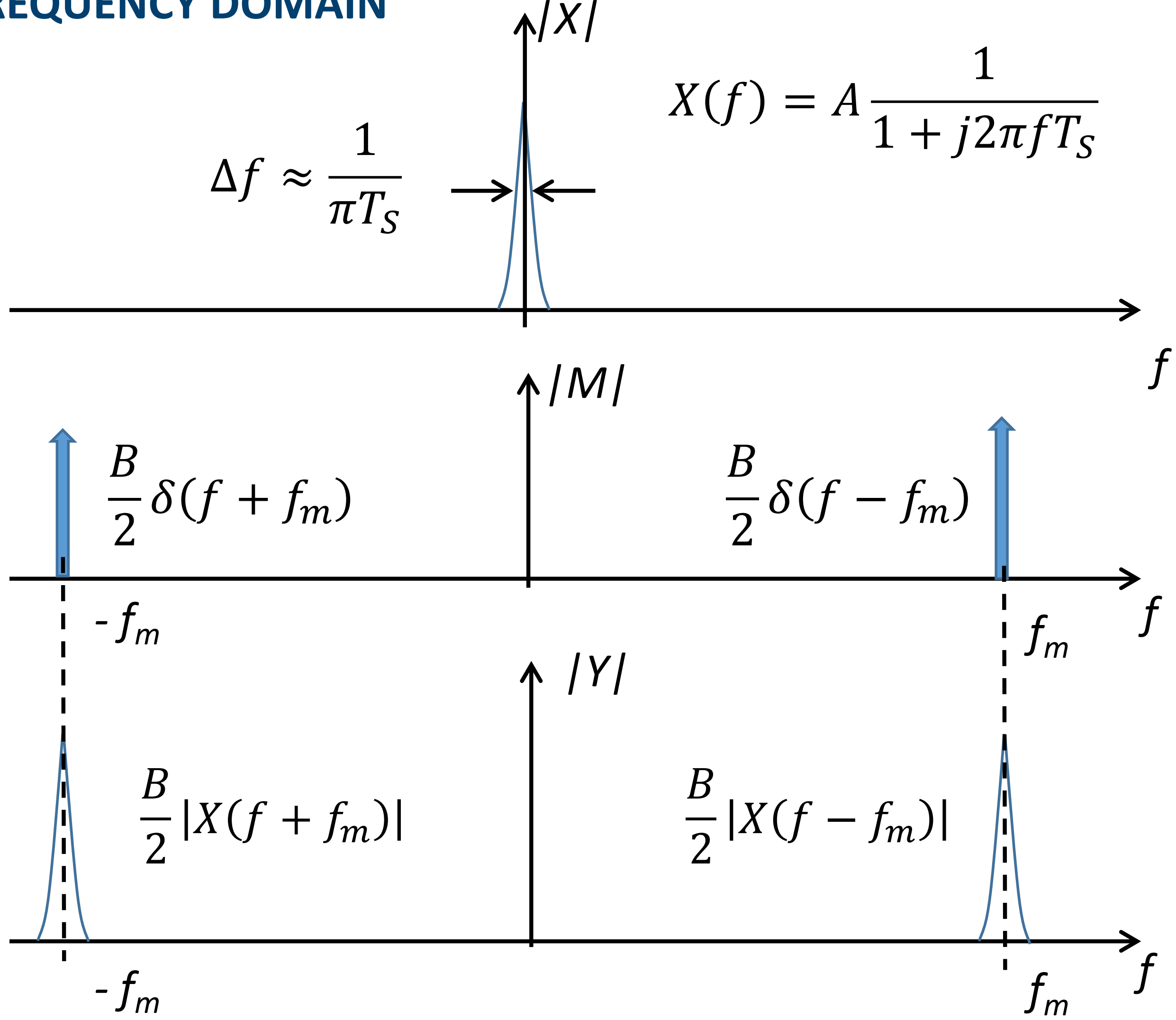
$$x(t) = 1(t) \cdot \frac{A}{T_S} e^{-\frac{t}{T_S}}$$

$m(t) = B\cos(2\pi f_m t + \varphi_m)$

with $f_m >> 1/T_S$

$y(t)=x(t) \cdot m(t)$

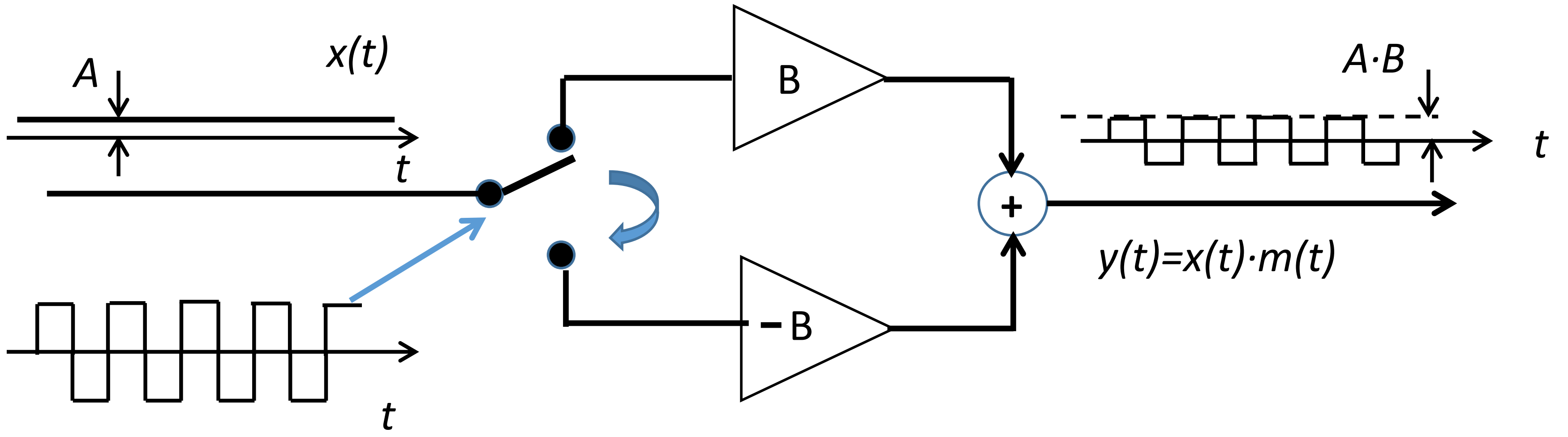
FREQUENCY DOMAIN



$Y(f) = X(f) * M(f)$

Squarewave Amplitude Modulation

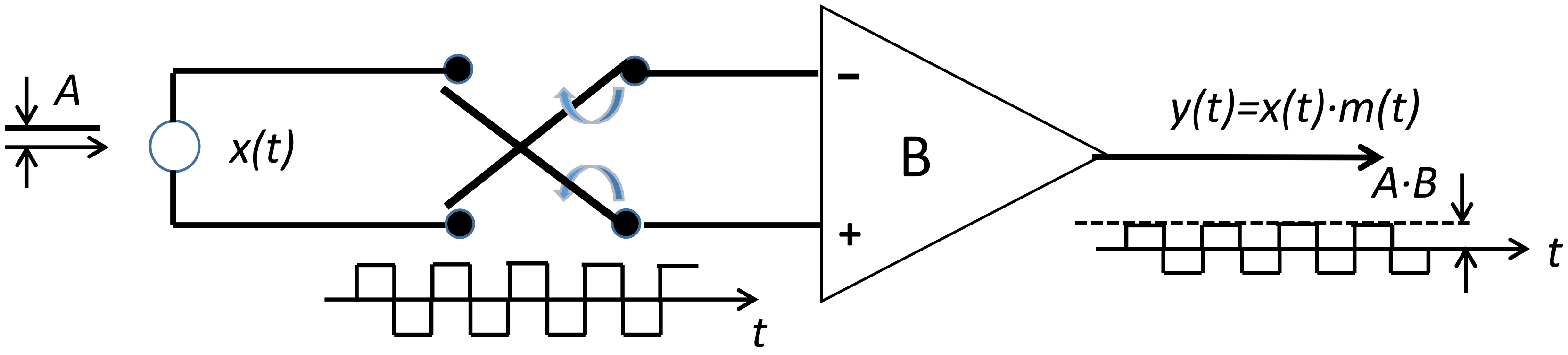
Modulation with a squarewave reference *m(t)* can be implemented with circuits based simply on switches and amplifiers, without analog multipliers



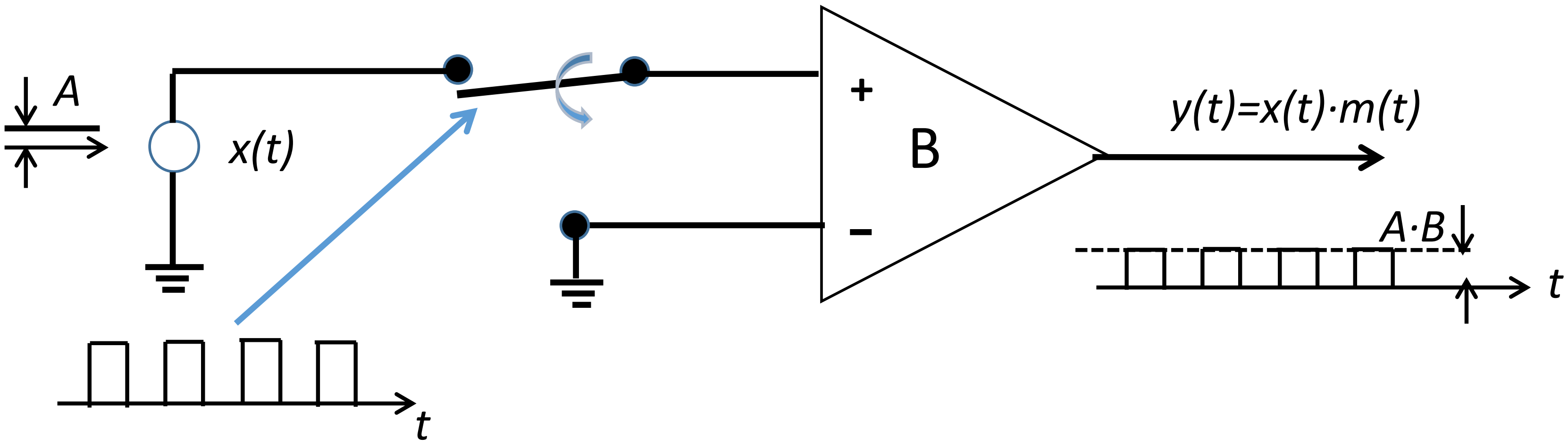
- In such cases, the circuit noise referred to the input is due mostly to the switch-devices and is much lower than that of analog multiplier circuits.
- **Metal-contact switches** have the lowest noise, but they can operate at limited switching frequency, typically up to a few 100Hz
- **Electronic switches** (MOSFET, diodes, etc.) operate up to very high frequencies but have fairly higher noise (anyway MOSFETs operating as switch-device have lower noise than MOSFETs operating as amplifier devices).

Squarewave Amplitude Modulation

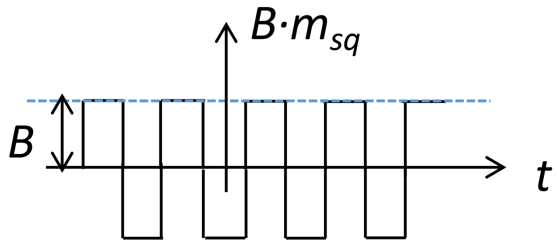
Switching example: differential amplifier with alternated input polarity



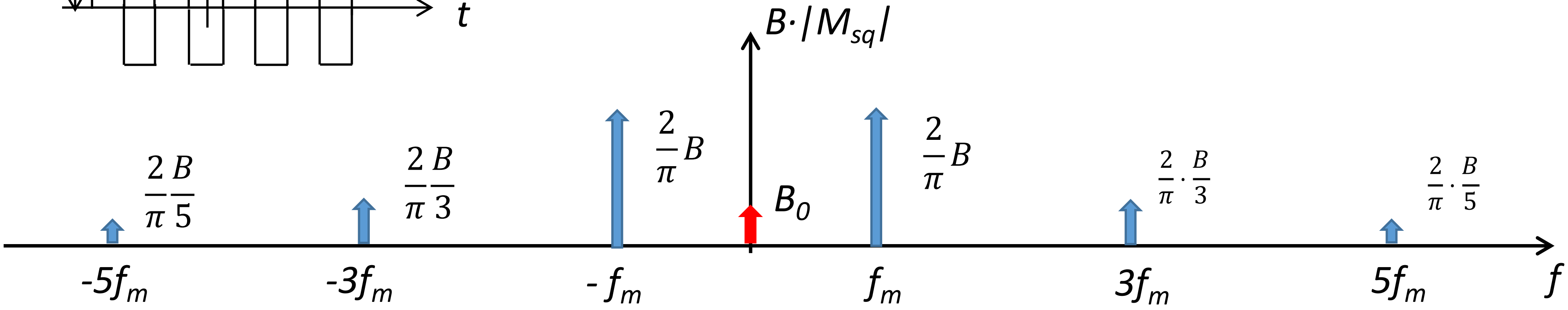
Switching example: chopper (ON-OFF modulation)



Squarewaves and F-transforms



$m_{sq}(t)$ = symmetrical squarewave (from +1 to-1) at frequency f_m



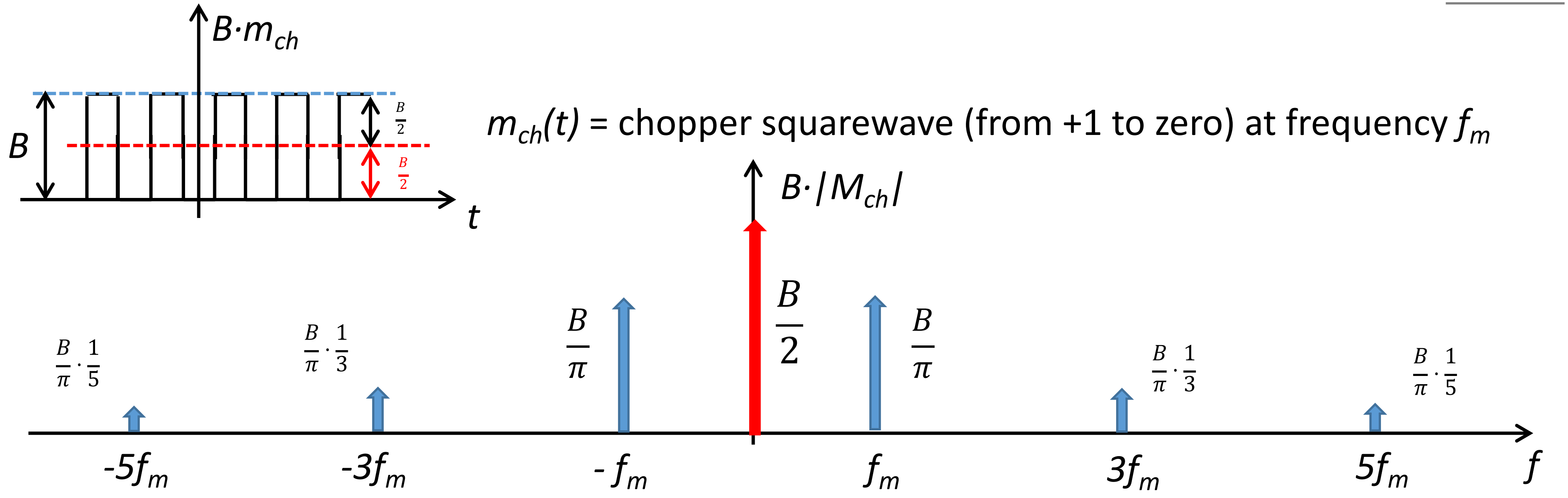
$$\boxed{B \cdot M_{sq}(f) = B \cdot \sum_{k=0}^{\infty} \frac{b_{2k+1}}{2}[\delta(f - f_{2k+1}) + \delta(f + f_{2k+1})]}$$

with $\begin{cases} f_{2k+1} = (2k+1)f_m \\ b_{2k+1} = \dfrac{(-1)^k}{(2k+1)} \cdot \dfrac{4}{\pi} \end{cases}$

In the amplitude modulation:

- each line of the reference M acts like a simple sinusoidal reference, i.e. shifts by its frequency and its phase the signal X and multiplies it by the amplitude $B \cdot b_{2k+1}$
- if the squarewave is not perfectly symmetrical (e.g. it has asymmetrical amplitude and/or duration of positive and negative parts) there is also a finite DC component with amplitude B_0 (possibly very small)
- the DC component does NOT transfer the signal X in frequency, just «amplifies» it by B_0

Squarewaves and F-transforms



$m_{ch}(t)$ = chopper squarewave (from +1 to zero) at frequency f_m

Chopper squarewave with amplitude B =

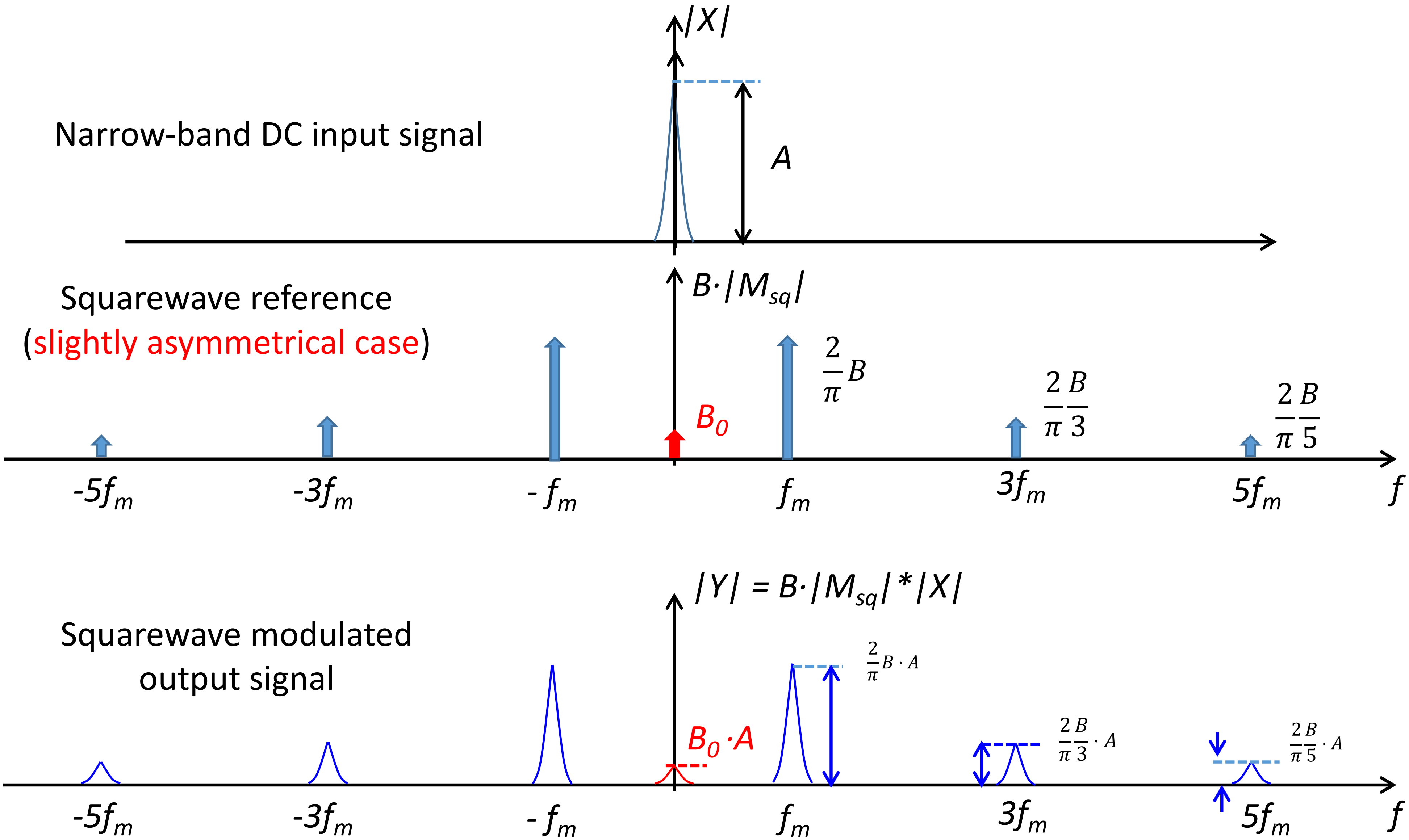
= Symmetrical squarewave with amplitude B/2 + DC component with amplitude B/2

$$\boxed{B \cdot M_{ch}(f) = \frac{B}{2} \cdot M_{sq}(f) + \frac{B}{2} \cdot \delta(f)}$$

In the amplitude modulation by a chopper:

- a replica of the signal X «amplified» by B/2 is transferred in frequency by the squarewave
- another replica of X «amplified» by B/2 is NOT transferred, it stays where it is

Squarewave Amplitude Modulation



Summary and Prospect

- As intuitive, narrow-band filtering is very effective for recovering narrow-band signals immersed in wide-band noise

- At low frequencies the 1/f noise added by the circuitry is overwhelming, so that the solution of narrow-band filtering becomes progressively less effective and finally insufficient for recovering signals with progressively lower frequency

- An effective approach to recover a low-frequency signal is to move it to higher frequency before the addition of 1/f noise. That is, to modulate the signal before the circuitry that contains the 1/f noise sources

- Narrow-band filtering can then be employed to recover the modulated signal; we will now proceed to analyze methods and circuits for narrow band filtering