COURSE OUTLINE

- Introduction
- Signals and Noise
- Filtering: OPF2 Optimum Filtering 2
- Sensors and associated electronics

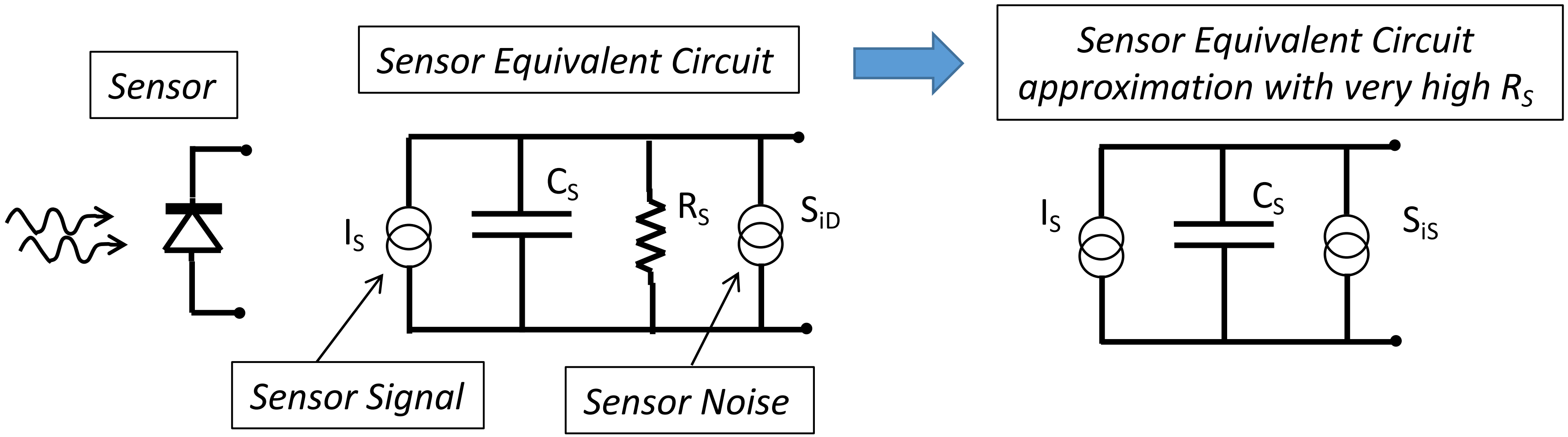
Optimum Filtering for High-Impedance Sensors

- High impedance sensors and low-noise preamplifiers
- Noise whitening filter
- Matched filter
- Practical approximations of the optimum filtering

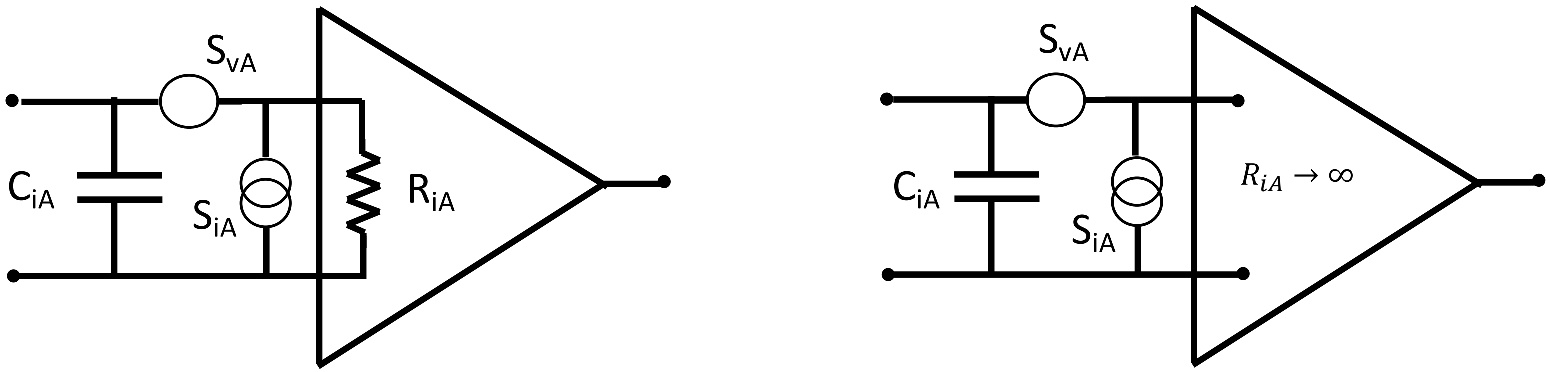
High Impedance Sensors and Low-Noise Preamplifiers

High-Impedance Sensors

- Let us consider sensors that are seen by the circuits connected to their terminals as generators of current signals with high internal impedance (typically a small capacitance C_S with a high resistance R_S in parallel)
- **Typical examples are**: p-i-n junction photodiodes and other photodetectors (CCDs, vacuum tube photodiodes, etc.); piezoelectric Force Sensors in quartz or other piezoelectric ceramic materials
- They have internal noise sources (e.g. shot current noise of a junction reverse current) modeled by a current noise generator in parallel to the signal generator



High-Impedance Sensors and Low-Noise Preamplifiers



Preamplifier equivalent circuit → *Approximation with very high R_{iA}*

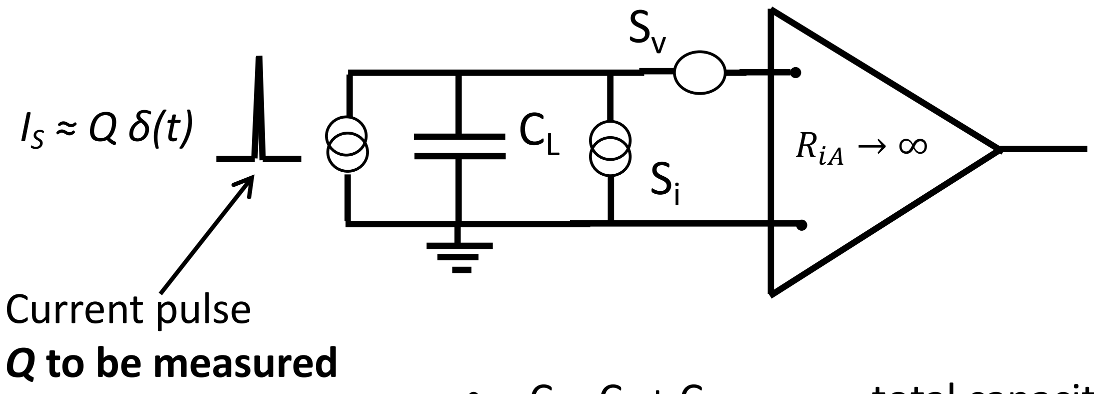
- R_{iA} = **true physical resistance** between the input terminals (**NOT the dynamic input resistance modified by the feedback in the amplifier**; e.g. not the low dynamic resistance of the virtual ground of an operational amplifier)
- Besides shot noise of bias currents, the S_{iA} includes Johnson resistor noise of R_{iA}

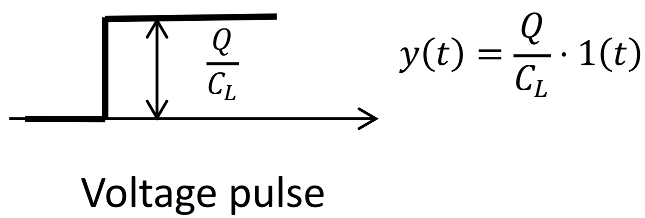
$$\boldsymbol{S_{iR} = \frac{4kT}{R_i}}$$

- The current noise directly faces the sensor current signal I_S **if R_{iA} is small the S_{iR} is overwhelming** (e.g. with $R_{iA} = 50\,\Omega$ it is $\sqrt{S_{iR}} \approx 18\, pA/\sqrt{Hz}$) and other components of $\sqrt{S_{iA}}$ are much lower (about $1\, pA/\sqrt{Hz}$ or lower)
- Conclusion: for **low-noise** operation of **high-impedance sensors**, it is **mandatory to employ a preamplifier with high input resistance R_{iA}**

High-Impedance Sensors and Low-Noise Preamplifiers

Equivalent circuit of high-impedance Sensor and Preamplifier
(approximation valid for **very high** sensor resistance $R_S \to \infty$)





- $C_L = C_S + C_{iA}$ total capacitance load
- $S_v = S_{vA}$ voltage noise generator (wideband white spectrum)
- $S_i = S_{iD} + S_{iA}$ current noise generator (wideband white spectrum)

At the preamplifier output:

- The voltage noise spectrum S_n has two components, it is **NOT white**

$$\boxed{S_n(\omega) = S_V + \frac{S_i}{\omega^2 C_L^2}}$$

- The voltage signal is a step with amplitude Q / C_L

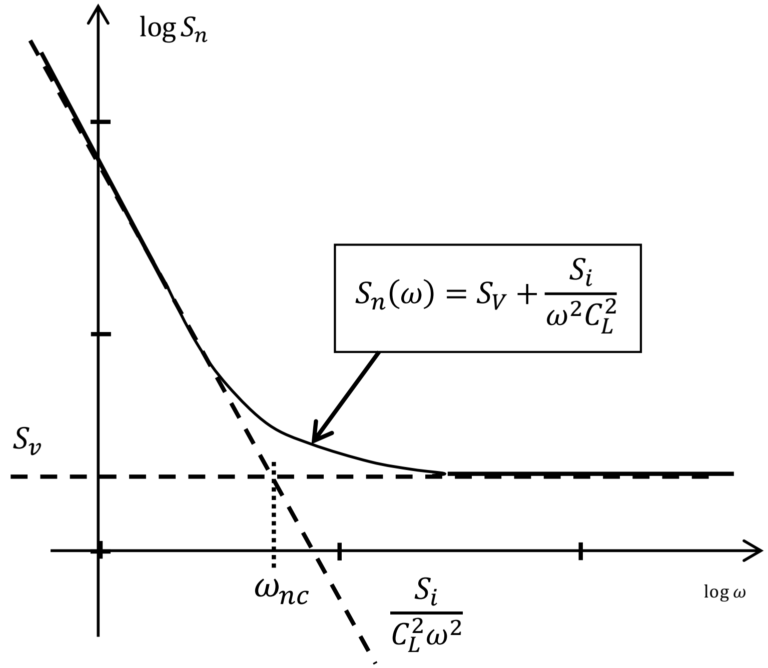
Noise Spectrum at Preamplifier Output

Crossing of the component defines
ω_{nc} Noise-Corner angular frequency

$$S_v = \frac{S_i}{C_L^2 \omega_{nc}^2} \Rightarrow \boxed{\omega_{nc} = \frac{\sqrt{S_i}}{C_L\sqrt{S_v}}}$$

$T_{nc} = 1/\omega_{nc}$ Noise-Corner time constant

$$\boxed{T_{nc} = \frac{1}{\omega_{nc}} = \frac{\sqrt{S_v}}{\sqrt{S_i}} C_L}$$



T_{nc} and ω_{nc} are fundamental parameters of the optimum filter: we will see that T_{nc} rules the duration of the filter weighting and ω_{nc} the filter bandlimit

We can define the **Noise Corner resistance** $\boxed{R_{nc} = \frac{\sqrt{S_v}}{\sqrt{S_i}}}$ so that $\boxed{T_{nc} = R_{nc} C_L}$

- with $\sqrt{S_v}$ a few $nV/\sqrt{Hz}$ and $\sqrt{S_i}$ ranging from a few 0,1 to 0,01 $pA/\sqrt{Hz}$
  R_{nc} ranges from tens to hundreds of kOhms
- with C_L from 0,1 pF to a few pF
  T_{nc} ranges from a few nanoseconds to some hundreds of nanoseconds

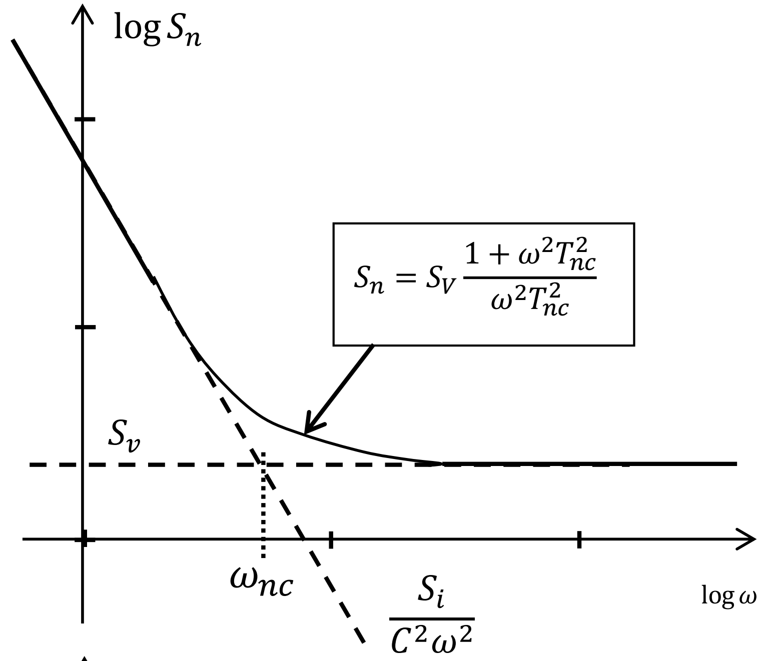
Noise whitening filter

Noise-Whitening Filter

The noise spectrum has

- a pole at $\omega_p = 0$
- a zero at $\omega_z = \omega_{nc} = 1/T_{nc}$

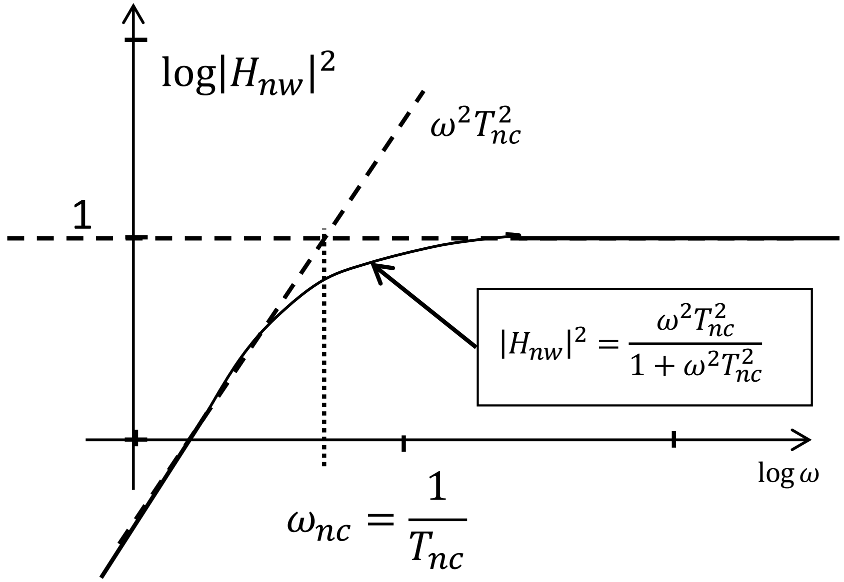
$$S_n(\omega) = S_V\left(1 + \frac{S_i}{\omega^2 S_V C_L^2}\right) = S_V\left(1 + \frac{1}{\omega^2 T_{nc}^2}\right) = S_V \frac{1 + \omega^2 T_{nc}^2}{\omega^2 T_{nc}^2}$$



The **noise whitening filter *H_{nw}*** must

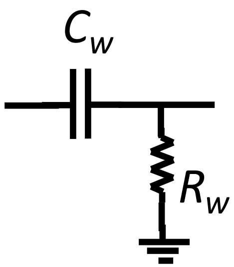
- cancel the pole with a zero at $\omega = 0$
- cancel the zero with a pole at $\omega = \omega_{nc} = 1/T_{nc}$

$$\boxed{|H_{nw}(\omega)|^2 = \frac{\omega^2 T_{nc}^2}{1 + \omega^2 T_{nc}^2}}$$



It is a simple high-pass filter

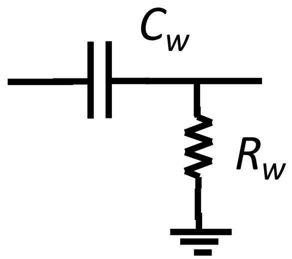
$$H_{nw}(\omega) = \frac{j\omega R_w C_w}{1 + j\omega R_w C_w}$$



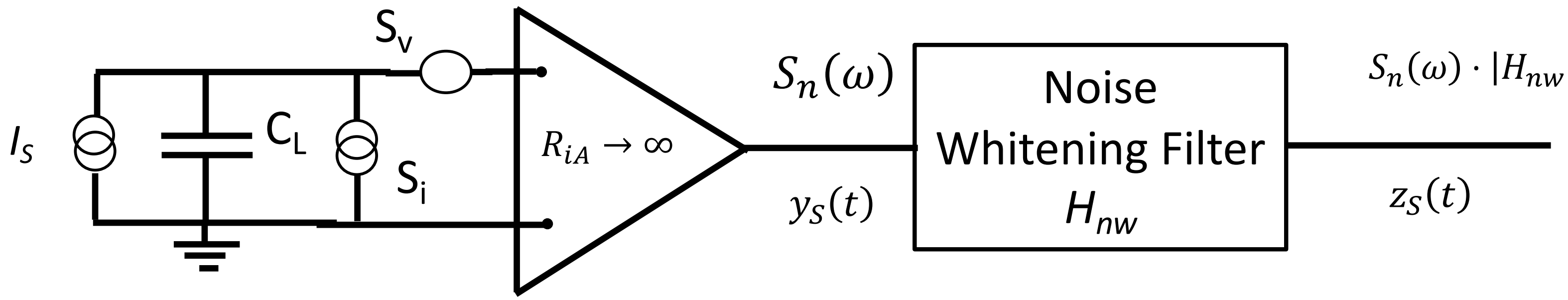
with $R_w C_w = T_{nc}$

Action of the Noise-Whitening Filter

Simple high-pass CR filter



$$H_{nw}(s) = \frac{sR_wC_w}{1 + sR_wC_w}$$ with $$R_wC_w = T_{nc}$$



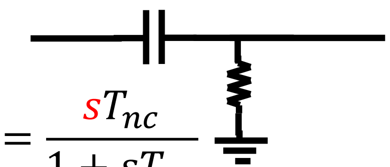
$$S_n(\omega) \cdot |H_{nw}(\omega)|^2 = S_v$$

it makes **white** the noise at its **output**

and changes the signal into a short **exponential pulse with time-constant *T_{nc}***

NW Filter

$$y_S(t) = \frac{Q}{C_L} \cdot 1(t)$$



$$H = \frac{sT_{nc}}{1 + sT_{nc}}$$

$$z_S(t) = \frac{Q}{C_L} \cdot 1(t) \cdot \exp\left(-\frac{t}{T_{nc}}\right)$$

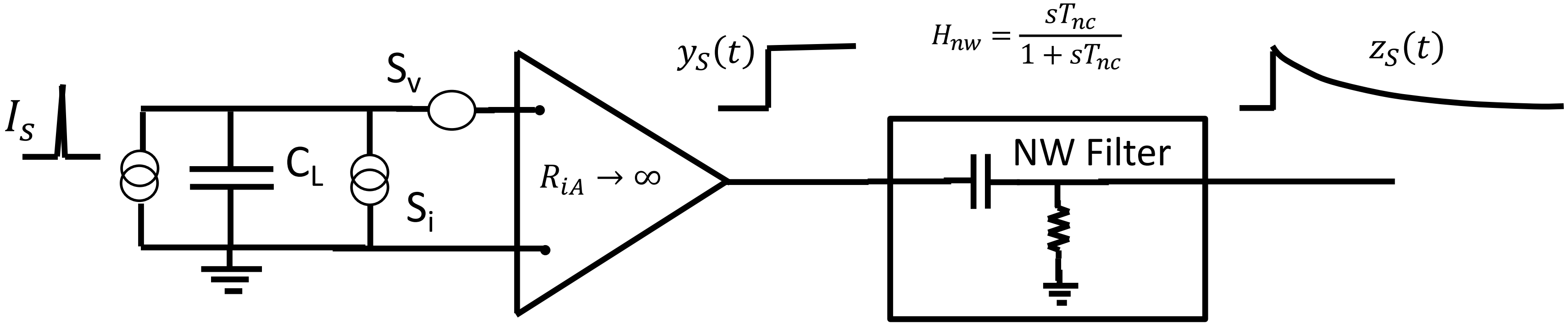
$$Y_S(s) = \frac{Q}{C_L} \cdot \frac{1}{s}$$

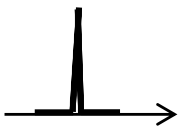
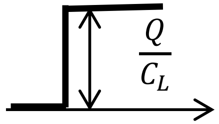
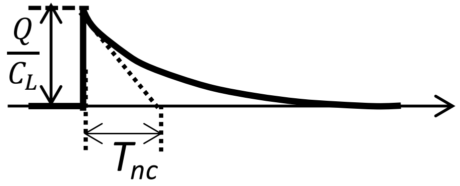
$$Z_S(s) = \frac{Q}{C_L} T_{nc} \cdot \frac{1}{1 + sT_{nc}}$$

Cancels pole at *s=0*

and replaces it by pole at *s = - 1/T_{nc}*

Signal at the output of the Noise-Whitening Filter

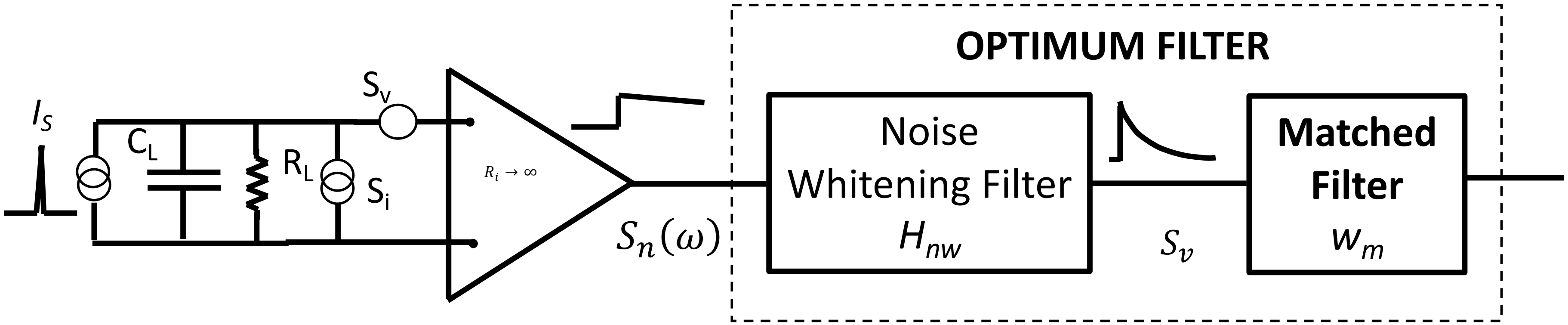


Input (current)	Preamp Output (voltage)	NW Filter Output (voltage)
δ – pulse 	Step pulse 	Exponential pulse 
$I_s(t) = Q \cdot \delta(t)$	$y_S(t) = \frac{Q}{C_L} \cdot 1(t)$	$z_S(t) = \frac{Q}{C_L} \cdot 1(t) \cdot \exp\left(-\frac{t}{T_{nc}}\right)$
$I_s(s) = Q$	$Y_S(s) = \frac{Q}{C_L} \cdot \frac{1}{s}$	$Z_S(s) = \frac{QT_{nc}}{C_L} \frac{1}{1+sT_{nc}}$

The **matched** filter has to be tailored to the signal at the whitening filter output, i.e. it must have **weighting function exponential with time constant T_{nc}**

Matched Filter

The Matched Filter completes the Optimum Filtering



In the case with finite load resistance R_L the whitening filter is different but the output signal produced is the same as with $R_L \rightarrow \infty$

$$z_S(t) = \frac{Q}{C_L} \cdot 1(t) \cdot \exp\left(-\frac{t}{T_{nc}}\right)$$

Therefore, the matched filter is the same in the two cases

$$\boxed{w_M = 1(t) \cdot \frac{1}{T_{nc}} \exp\left(-\frac{t}{T_{nc}}\right)}$$

and gives the same result

$$\left(\frac{S}{N}\right)^2_{opt} = \frac{A^2}{S_v} \cdot \int_{-\infty}^{\infty} b^2(\alpha)\, d\alpha \quad \Rightarrow \quad \left(\frac{S}{N}\right)^2_{opt} = \frac{Q^2}{C_L^2} \frac{T_{nc}^2}{S_v} \int_{-\infty}^{\infty} w_m^2\,(\alpha) d\alpha = \frac{Q^2}{C_L^2} \frac{1}{2} \frac{T_{nc}}{S_v}$$

Optimum Filtering

At the output of the optimum filter (i.e. of the matched filter) we have

Signal $$s_o = \int_0^\infty z_s(\alpha)\, w_m(\alpha) d\alpha = \frac{Q}{C_L}\frac{1}{T_{nc}}\int_0^\infty \exp\left(-\frac{2\alpha}{T_{nc}}\right) d\alpha = \frac{1}{2}\frac{Q}{C_L}$$

Noise $$\sqrt{\overline{n_o^2}} = \sqrt{S_v}\cdot\sqrt{k_{ww}(0)} = \sqrt{S_v}\frac{1}{T_{nc}}\sqrt{\int_0^\infty \exp\left(-\frac{2\alpha}{T_{nc}}\right) d\alpha} = \sqrt{\frac{S_v}{2T_{nc}}}$$

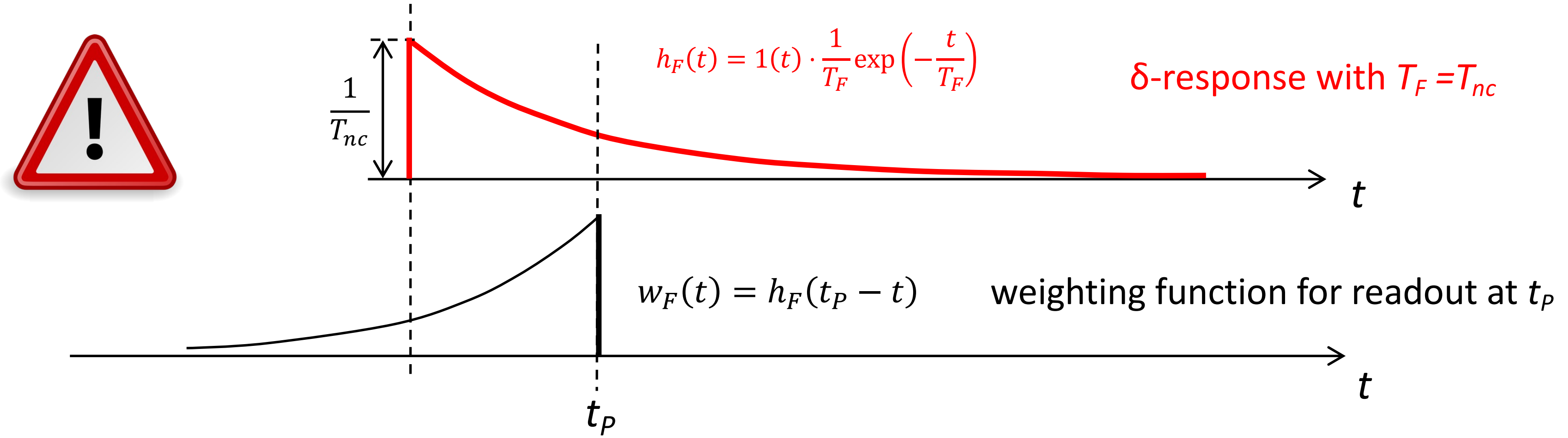
S/N $$\boxed{\frac{s_o}{\sqrt{\overline{n_o^2}}} = \frac{1}{2}\frac{Q}{C_L}\sqrt{\frac{2T_{nc}}{S_v}}}$$

- The optimum filter theory specifies **what is the best S/N physically obtainable** and the kind of filter required for attaining it
- The optimum filter can be implemented in reality, but the filter design turns out to be quite **complex**
- Furthermore, the optimum filter takes **infinite time** (after the signal onset) to complete its action, which is not acceptable in practice
- It is possible, however, to consider and evaluate fairly simple filters that **approximate the optimum** filter and closely approach its performance

Practical approximations of the optimum filtering

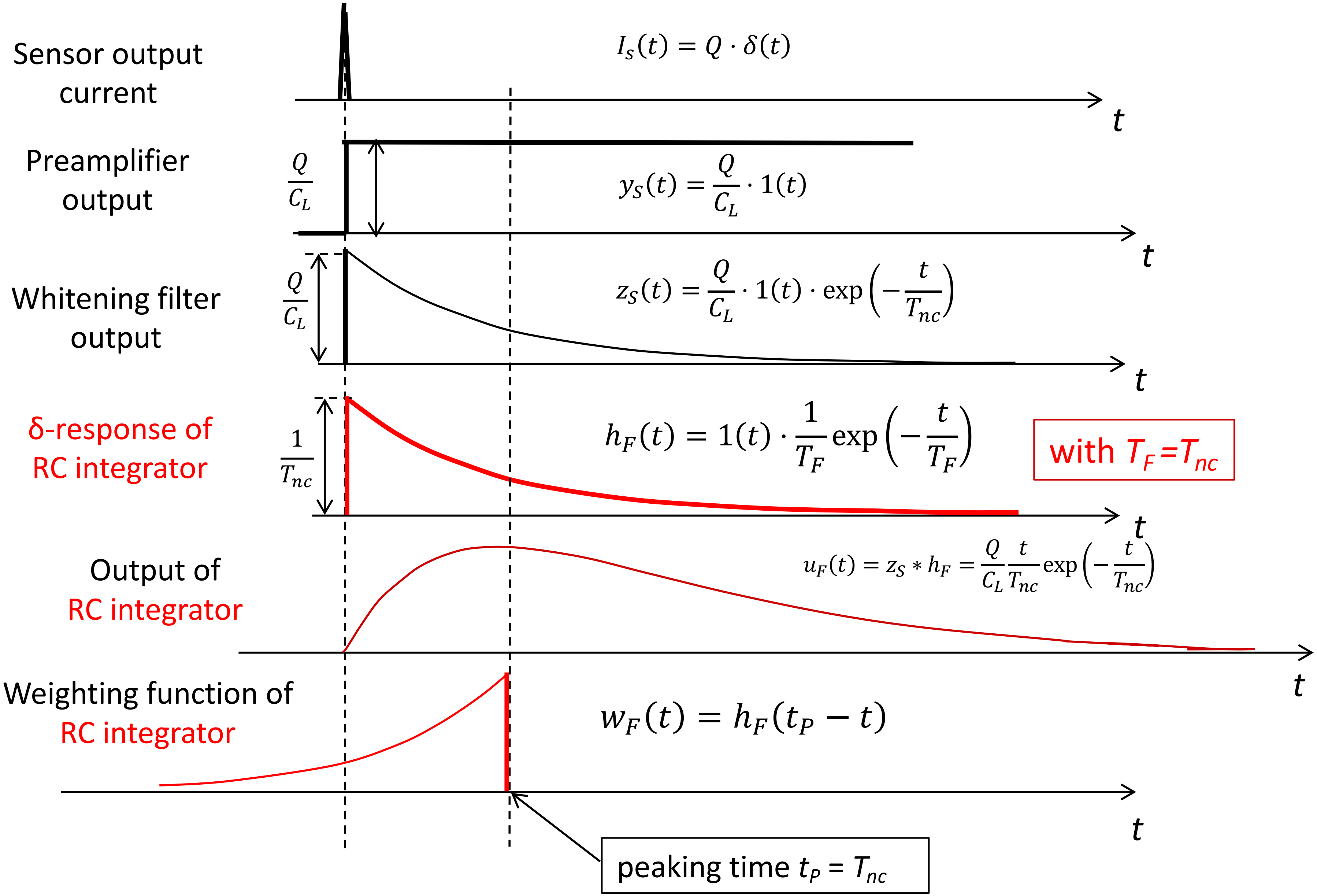
RC integrator Approximation of the Matched Filter

- The features of the matched filter weighting function observed in time and in frequency point out that it is a **low-pass filter**
- A simple **RC integrator** (single-pole low-pass filter) can be an **approximation** of the matched filter. With $RC = T_{nc}$ its δ-response $h_F(t)$ is identical to the weighting function $w_M(t)$ of the matched filter.
- **The RC weighting $w_F(t)$ has the same shape as $w_M(t)$ of the matched filter, but it's not fully correct because it is reversed in time!**



- **Noise** filtering is **equal to the matched filter**, since it is unaffected by time-inversion; the output is white noise with band-limit set by a simple pole with time-constant T_F .
- **Signal** filtering is **different** from the matched filter, since it is modified by time-inversion

RC integrator Approximation of the Matched Filter



RC integrator Approximation compared to the Optimum Filter

The RC output signal waveform is $$u_F(t) = \frac{Q}{C_L}\frac{t}{T_{nc}}\exp\left(-\frac{t}{T_{nc}}\right)$$

Signal peak value (at $t = T_{nc}$) $$s_F = u_F(T_{nc}) = \frac{1}{e}\frac{Q}{C_L}$$

Noise $$\sqrt{\overline{n_F^2}} = \sqrt{S_v}\cdot\sqrt{k_{hh}(0)} = \sqrt{\frac{S_v}{2T_{nc}}}$$

S/N $$\frac{s_F}{\sqrt{\overline{n_F^2}}} = \frac{1}{e}\frac{Q}{C_L}\sqrt{\frac{2T_{nc}}{S_v}}$$

Comparing the RC approximation with the ideal optimum filter system we see that

$$s_F = \frac{2}{e}s_o \approx 0{,}736\cdot s_o$$ the signal is lower

$$\sqrt{\overline{n_F^2}} = \sqrt{\overline{n_o^2}}$$ the noise is equal

$$SNR_F = \frac{2}{e}\eta_o \approx 0{,}736\cdot SNR_o$$ the S/N is lower

the performance of the filter system with RC approximation of matched filter is about 27% worse than the absolute optimum.
Note that the loss is due to bad exploitation of the signal