COURSE OUTLINE

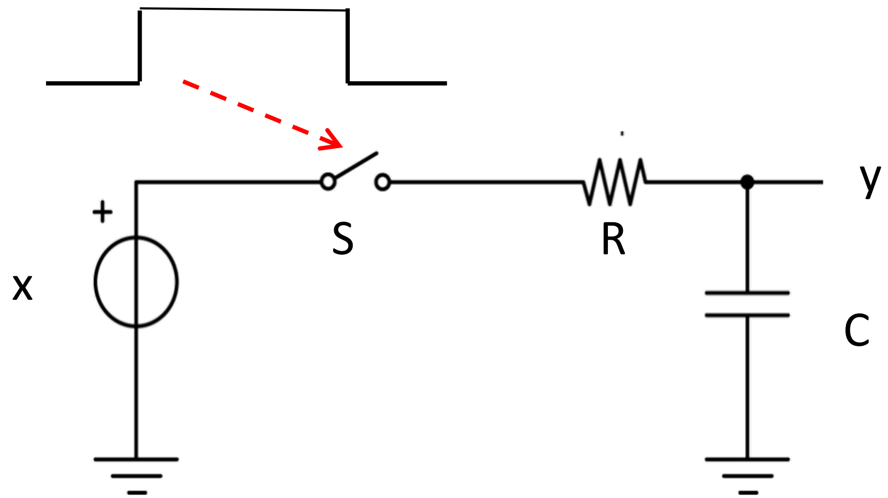
- Introduction
- Signals and Noise
- **Filtering: LPF2 Switched-Parameter Filters**
- Sensors and associated electronics

Switched-parameter filters

- Switched-parameter RC low-pass filters
- Sample and Hold S&H
- Gated Integrator GI

Switched-parameter RC low-pass filters

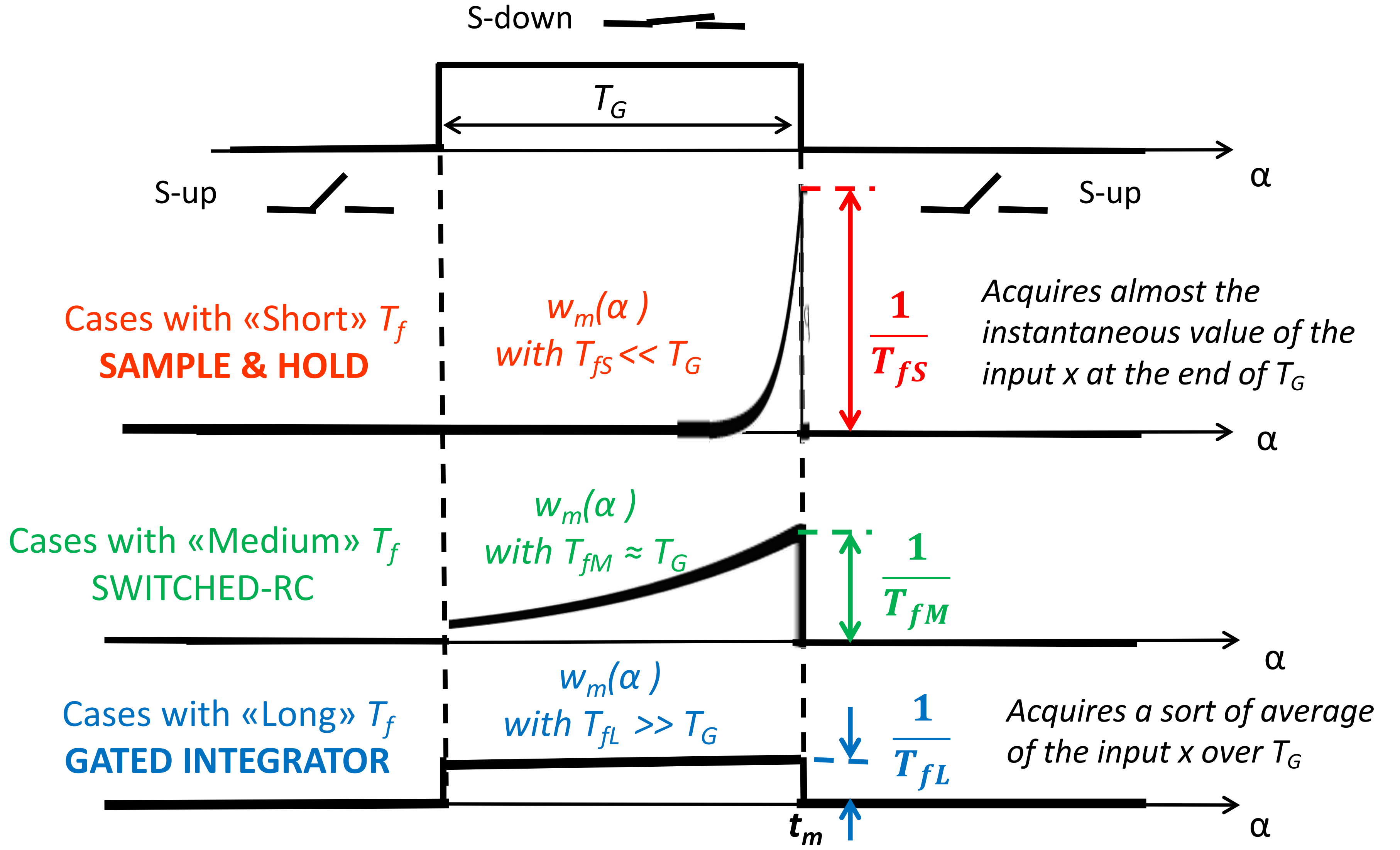
Switched-parameter RC low-pass filters



- **State with *S* down (closed in short circuit):** the circuit behaves like a constant-parameter RC integrator; current can flow in and out of C
- **State with *S* up (open circuit):** the circuit is in HOLD, no current can flow, the charge previously stored in C is maintained, the voltage on C stays constant.

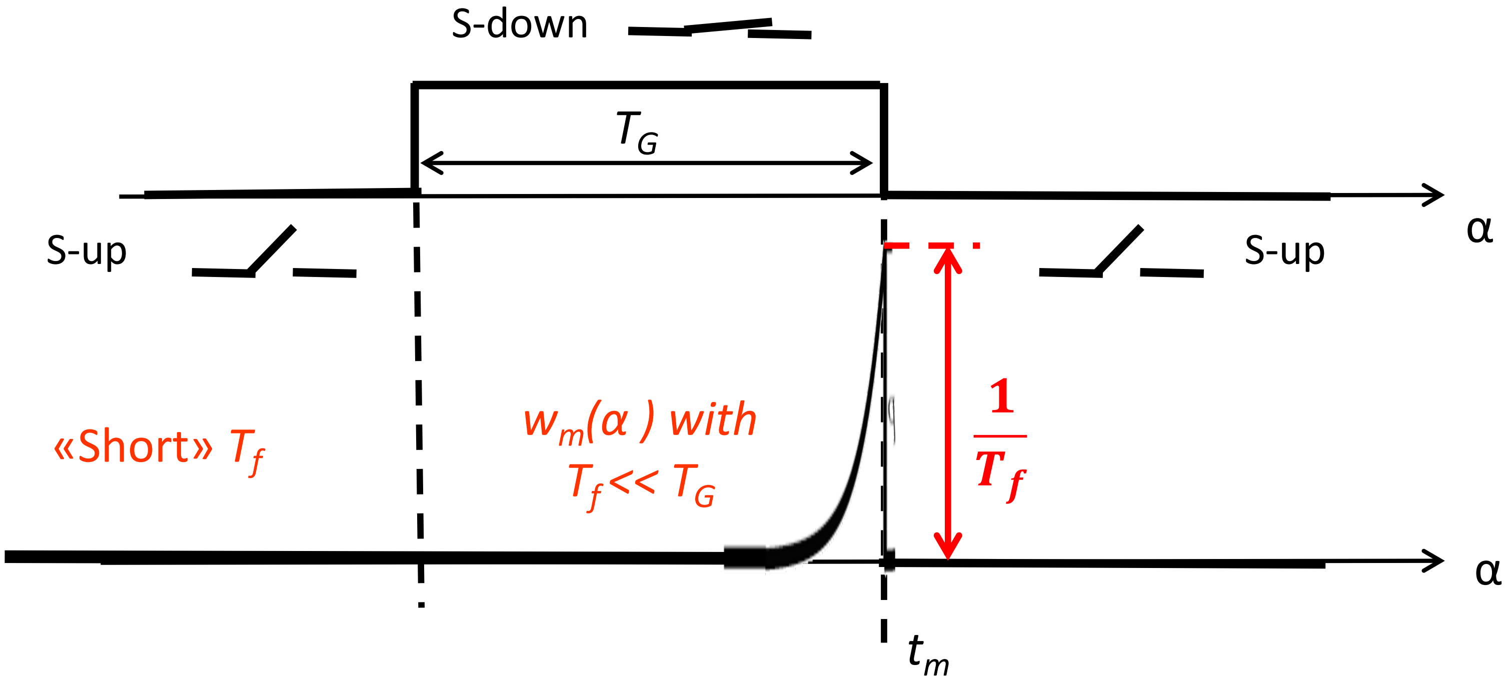
In the cases here considered:
(a) the initial state is with *S* open and zero charge in C
(b) the command closes *S* in synchronism with the signal to be acquired
and re-opens S after the acquisition

Switched-parameter RC low-pass filters



Sample and Hold S&H

Sample and Hold (S&H)



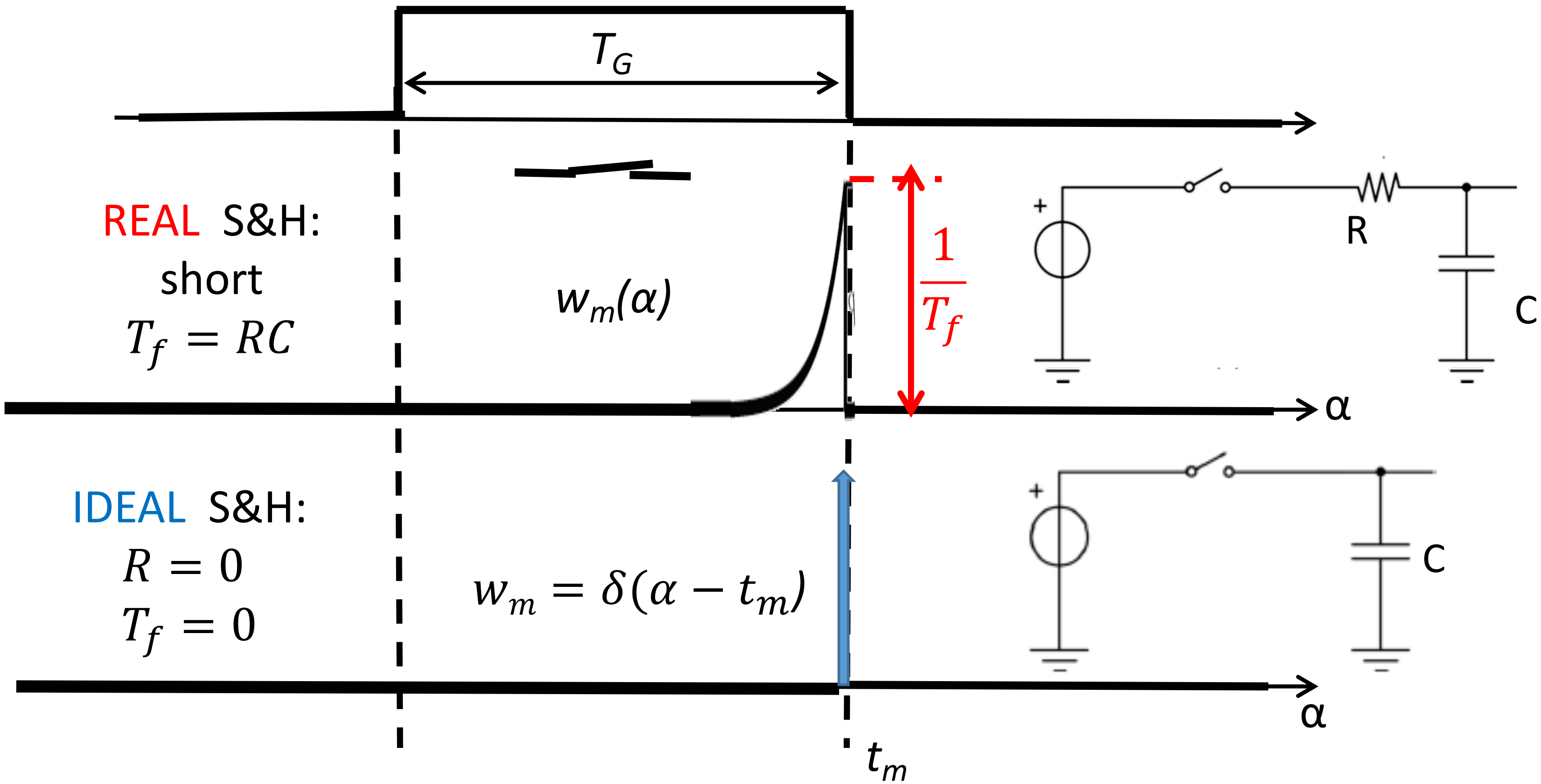
The S&H has **unity DC gain** (C is fully charged at the input voltage within T_G)

$$W_m(0) = \int_0^\infty w_m(\alpha) d\alpha = 1$$

The S&H has very mild filtering action, equivalent to that of a constant-parameter RC integrator with equal time constant T_{fS} . With wide-band input noise S_b (bilateral)

$$\overline{y_n{}^2} = S_b \cdot \frac{1}{2T_f}$$

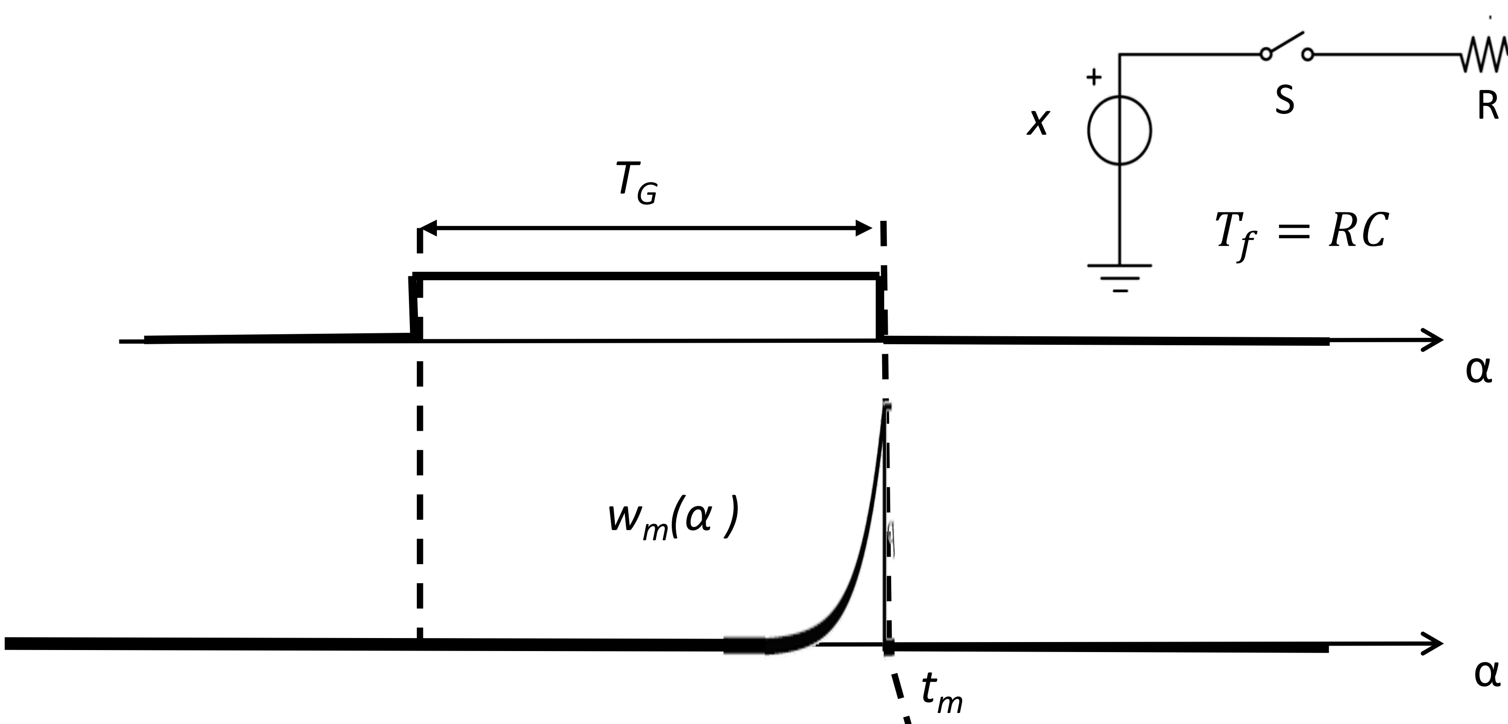
Real vs ideal S&H



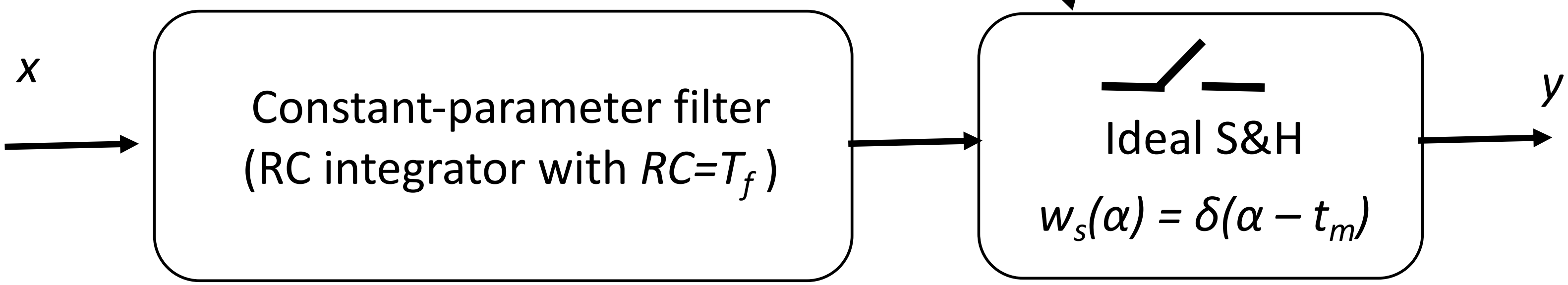
- The **minimum available T_f is limited by the technology** of devices and circuits (finite R values of fast switching devices and C values required for holding information)

- **S&H acquisition time** = time for reaching the full output value ≈ a few T_f , i.e. currently
  some tens of nanoseconds in discrete-component circuits
  some tens of picoseconds in integrated circuits with minimized capacitances

S&H equivalent model



The output of a real S&H is equivalent to (and can be modeled as) the cascade of two stages:



S&H Readout Noise

- **READOUT NOISE** of a sampling circuit is the contribution to the output noise due to the internal noise sources in the sampling circuit itself
- In the S&H the main source of readout noise is the wide-band Johnson noise of R with spectral density $S_{bB} = 2kTR$ (bilateral)

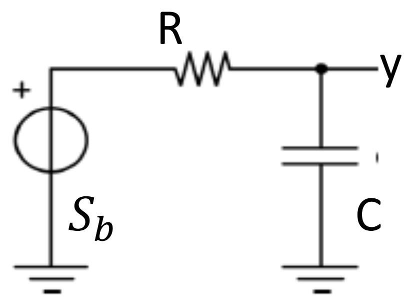
Since

$$w(\alpha) = \frac{1}{T_f} e^{-\frac{(t_m-\alpha)}{T_f}} 1(t_m - \alpha) \quad \text{and} \quad k_{ww}(\tau) = \frac{1}{2T_f} e^{-\frac{|\tau|}{T_f}}$$

the readout noise is

$$\overline{y_R{}^2} = S_{bB} \cdot k_{ww}(0) = 2kTR \cdot \frac{1}{2T_f} = 2kTR \cdot \frac{1}{2RC}$$

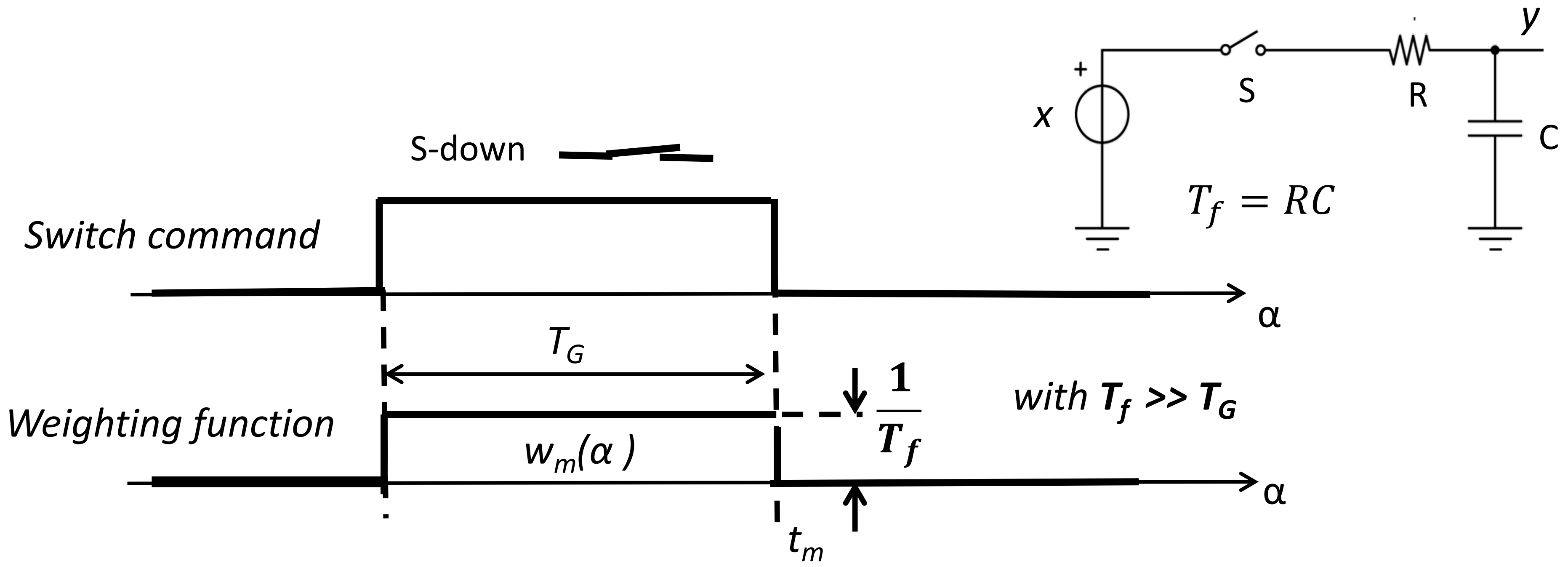
$$\boxed{\overline{y_R{}^2} = \frac{kT}{C}}$$



this is just the noise generated and self-filtered by a constant parameter RC filter and is INDEPENDENT OF THE R VALUE, in agreement wth the S&H circuit model.

Gated Integrator GI

Gated Integrator (GI)



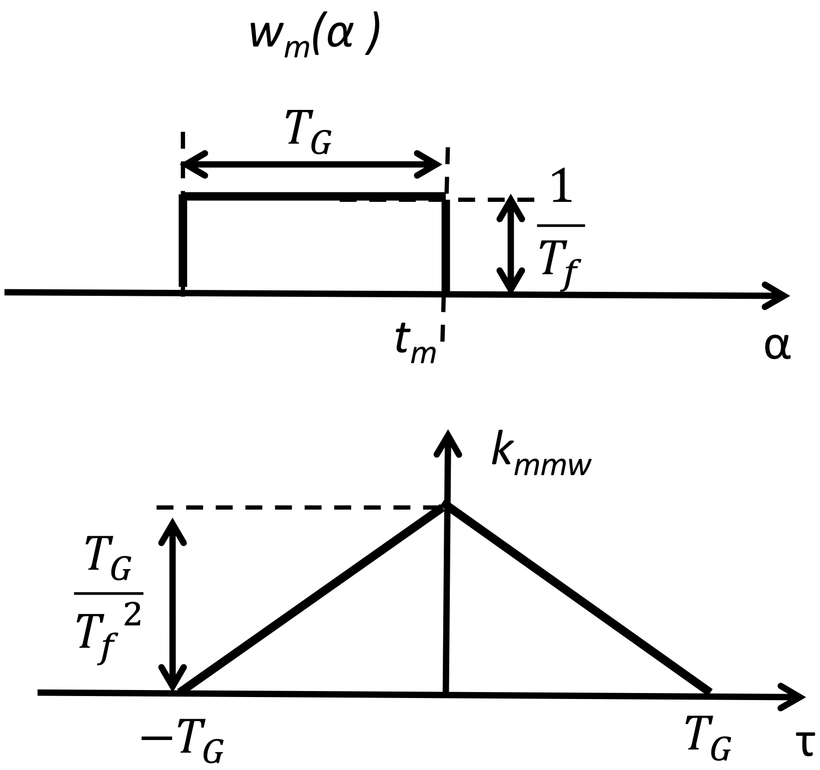
- For behaving as GI (uniform weight in T_G) the circuit must have $T_f >> T_G$
- Therefore, the **DC gain G is inherently much less than unity**

$$G = W_m(0) = \int_0^{\infty} w_m(\alpha) d\alpha = \frac{T_G}{T_f} \ll 1$$

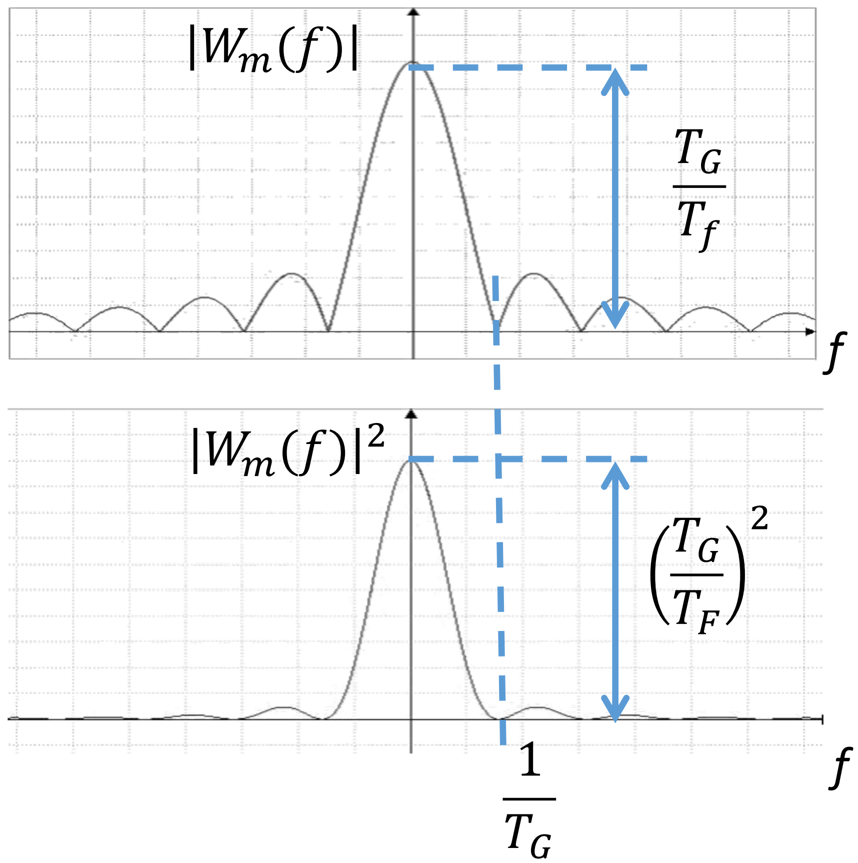
- A GI has remarkable filtering action on a wide-band input noise, that is, on noise with autocorrelation width much shorter than the gate duraton T_G .
- Long gate duration T_G is well feasible in practice, much better than a long averaging interval T_a in a mobile-mean filter

Gated Integrator (GI)

TIME DOMAIN



FREQUENCY DOMAIN



Filtering and S/N enhancement by GI

INPUT:

- signal x_s constant in T_G (DC signal)
- wide-band noise S_b (bandwidth $f_n >> 1/T_G$ and autocorrelation width $T_n << T_G$)

$$\overline{x_n^2} = S_b 2 f_n = S_b / 2T_n$$

OUTPUT:

Signal $y_s = x_s \cdot \frac{T_G}{T_f} = x_s G$ i.e. with gain

$$\boxed{G = \frac{T_G}{T_f} << 1}$$

Noise
$$\overline{y_n^2} = S_b \cdot \frac{T_G}{T_f^2} = \frac{S_b}{T_G} \cdot \left(\frac{T_G}{T_f}\right)^2 = \frac{S_b}{T_G} \cdot G^2 =$$

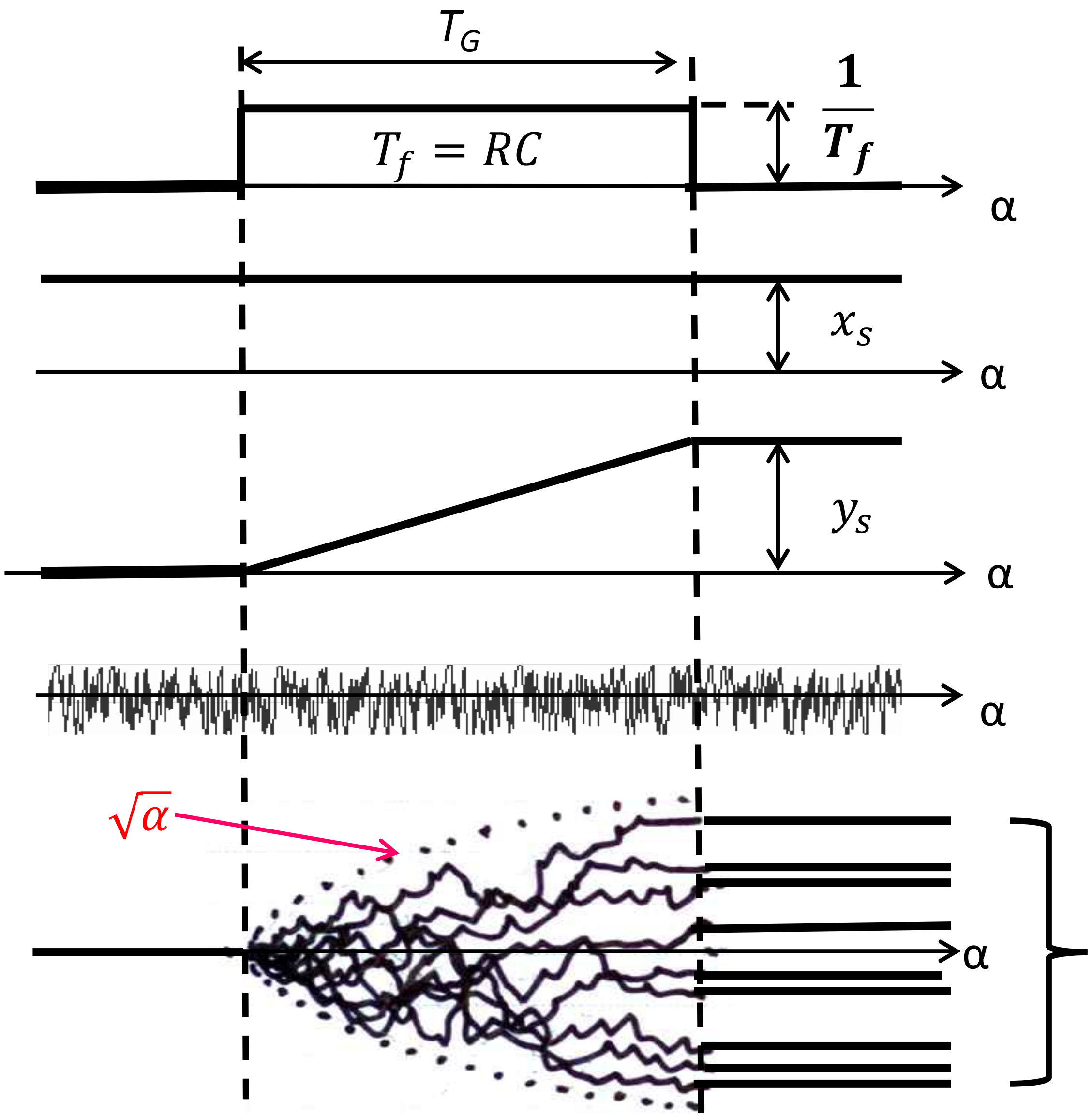
$$= \frac{S_b}{2T_n} \frac{2T_n}{T_G} G^2 = \overline{x_n^2} \cdot \frac{2T_n}{T_G} \cdot G^2$$

Signal-to-noise ratio

$$\left(\frac{S}{N}\right)_y = \frac{y_s}{\sqrt{\overline{y_n^2}}} = \frac{x_s}{\sqrt{\overline{x_n^2}}} \cdot \sqrt{\frac{T_G}{2T_n}} = \left(\frac{S}{N}\right)_x \cdot \sqrt{\frac{T_G}{2T_n}}$$

NB: the output signal increases as T_G and the noise as $\sqrt{T_G}$, therefore the S/N increases as the square root of the gate time $\sqrt{T_G}$

Output Signal and Noise of GI



weighting function $w_m(\alpha)$

Input DC signal x_s

Output signal $y_s = x_s \cdot \frac{T_G}{T_f} \propto \boldsymbol{T_G}$

Wide-band input noise x_n

Output noise

$$\sqrt{\overline{y_n^{\,2}}} = \sqrt{S_b \cdot \frac{T_G}{T_f^{\,2}}} \propto \sqrt{\boldsymbol{T_G}}$$

GI compared to other LPF

Fair comparison between different LPF with different DC gain G can be made by considering the value of the **filtered noise referred to the input** of the filter (and the input signal). This is equivalent to consider the **output with unity DC gain** (if necessary, by considering to add further gain stages).

For a GI this noise is

$$\left(\overline{x_n{}^2}\right)_{GI} = \frac{\left(\overline{y_n{}^2}\right)_{GI}}{G^2} = \frac{S_b}{T_G}$$

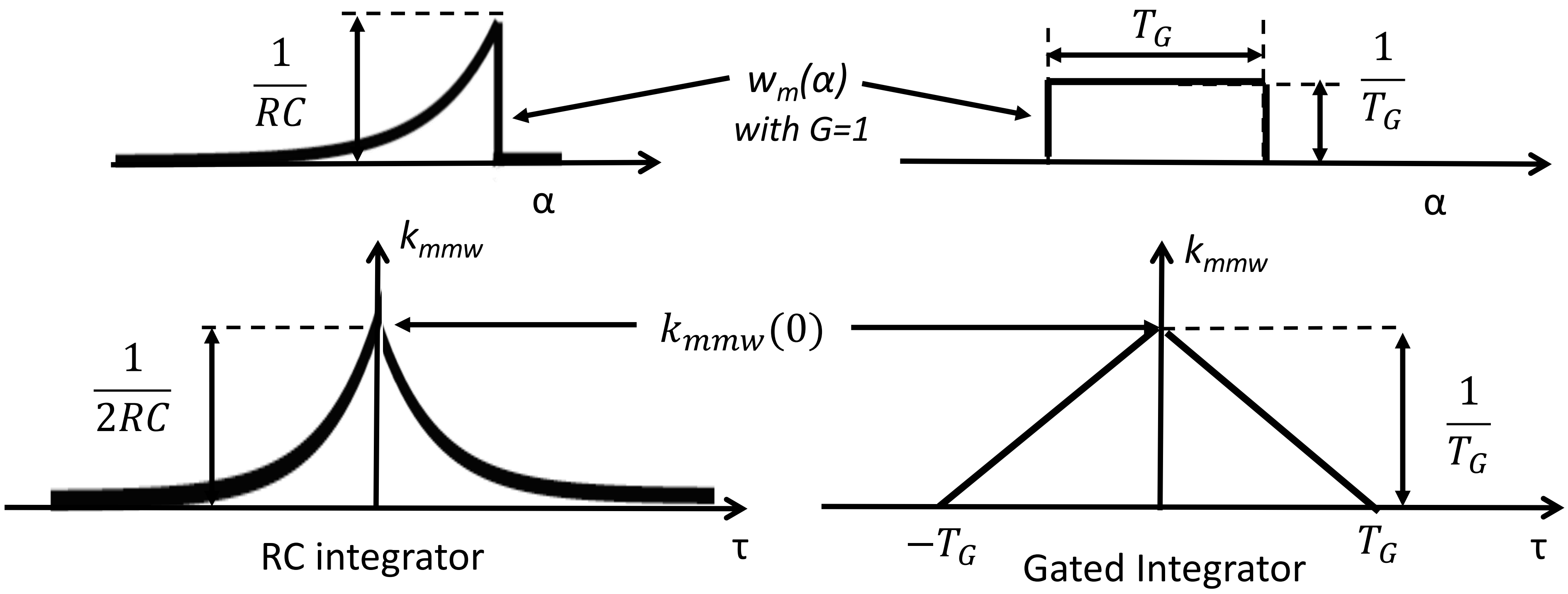
For a constant-parameter RC (inherently with G=1) that filters the same wide-band noise S_b it is

$$\left(\overline{x_n{}^2}\right)_{RC} = \left(\overline{y_n{}^2}\right)_{RC} = \frac{S_b}{2RC}$$

Therefore, as concerns the S/N obtained for input DC signals accompanied by wide-band noise, GI and RC integrator are equivalent if

$$T_G = 2RC$$

GI and equivalent RC-integrator



We consider here filters with **equal DC gain of unity**, hence with equal output signal.

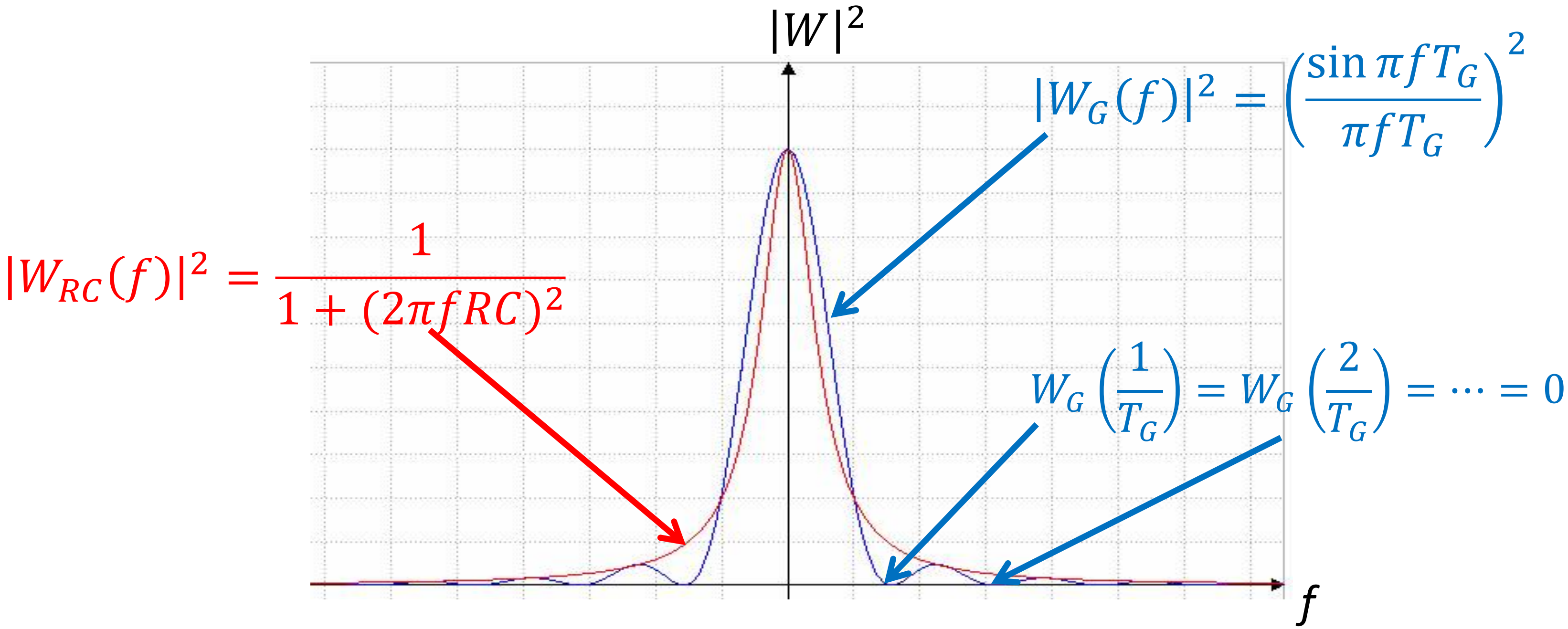
With wide-band input noise S_b the output noise is

$$\overline{y^2} = S_b \cdot k_{mmw}\ (0)$$

therefore, GI and RC have **equal output noise** if

$$\boxed{T_G = 2R\,C}$$

GI and equivalent RC-integrator



With $T_G = 2RC$ they are equivalent for:

- the S/N obtained with wide-band noise and DC signal input
- the attenuation of high-frequency disturbances in general

However:

- **The GI has zeros of $W_G(f)$ at $f_k = k/T_G$ that can be exploited to cancel specific disturbances at known frequencies (radio frequencies or mains frequency and harmonics)**