COURSE OUTLINE

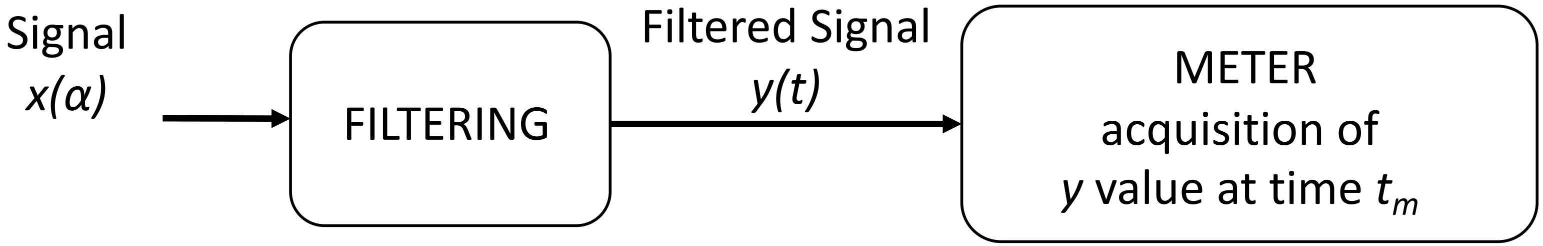
- Introduction
- Signals and Noise
- **Filtering Signals**
- Sensors and associated electronics

Filtering Signals

- Discrete-Time and Continuous-Time Signal Filtering
- Filter Weighting Function in the Time Domain
- Linear Filters with Constant-Parameters
- Linear Filters with Variant-Parameters
- Time-Variant Weighting Function in the Frequency-Domain

Discrete-Time and Continuous-Time Signal Filtering

Signal filtering and acquisition



- **Linear filtering** = the superposition of effects is valid
- The output is a weighted sum of input values *x* taken at various times *α* with **weights that do NOT depend on the input x**

- **We start choosing t_m and then the α_1, α_2, α_n times of analysis (e.g. 5s before, 4s..)**

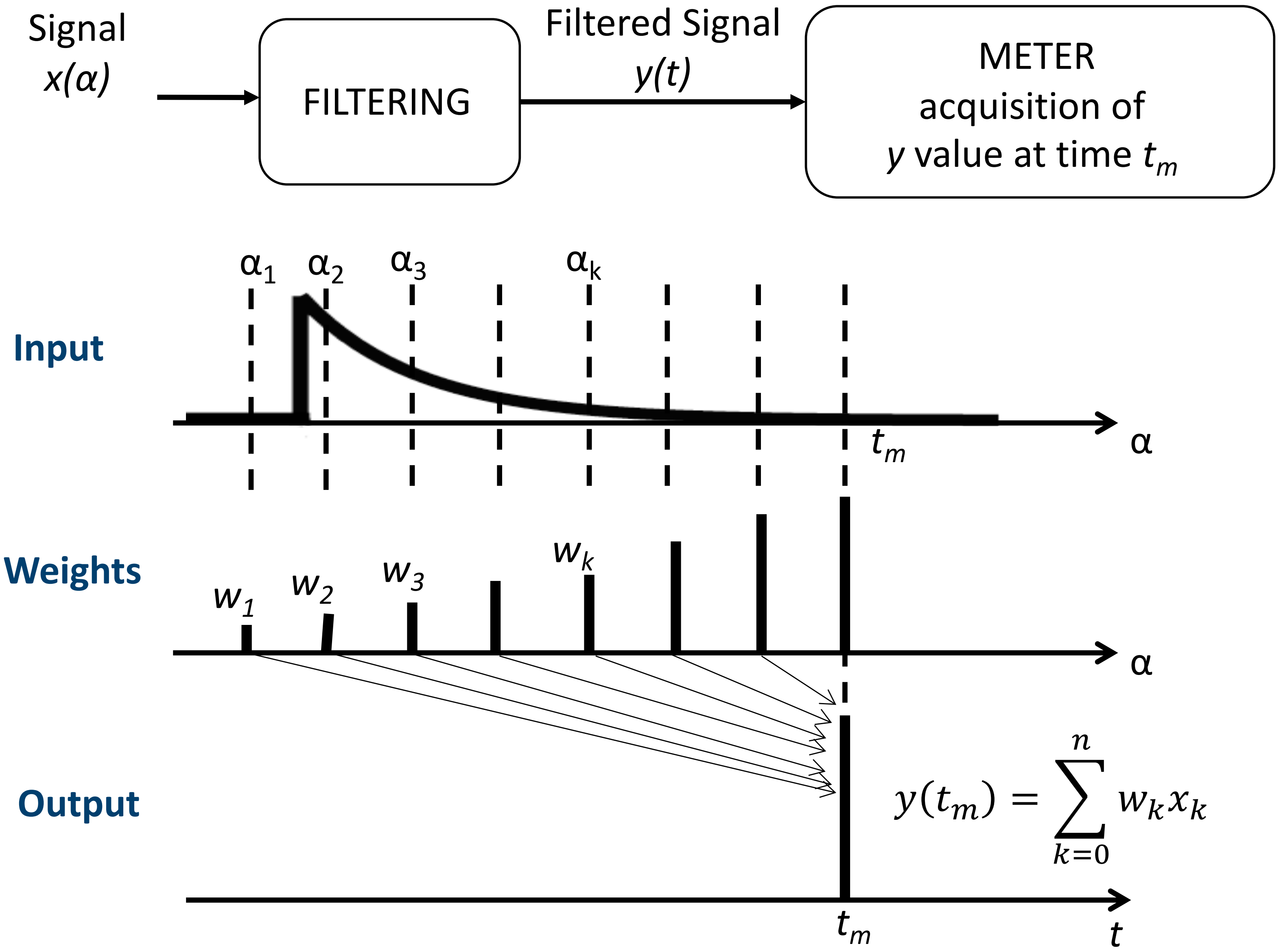
In **discrete-time** filtering (*e.g. in digital signal processing DSP*)

$$y(t_m) = w_1 x(\alpha_1) + w_2 x(\alpha_2) + w_3 x(\alpha_3) + \cdots + w_n x(\alpha_n) = \sum_{k=0}^{n} w_k x(\alpha_k)$$

$$y(t_m) = \sum_{k=0}^{n} w_k x_k$$

If the weights are the **same set w_k for any t_m** (for any acquisition time)
it is a **constant-parameter** filtering ; otherwise, it is a time-variant filtering

Discrete-time Signal filtering



$$y(t_m) = \sum_{k=0}^{n} w_k x_k$$

Continuous-time Signal filtering

instead of a sum of **discrete values**

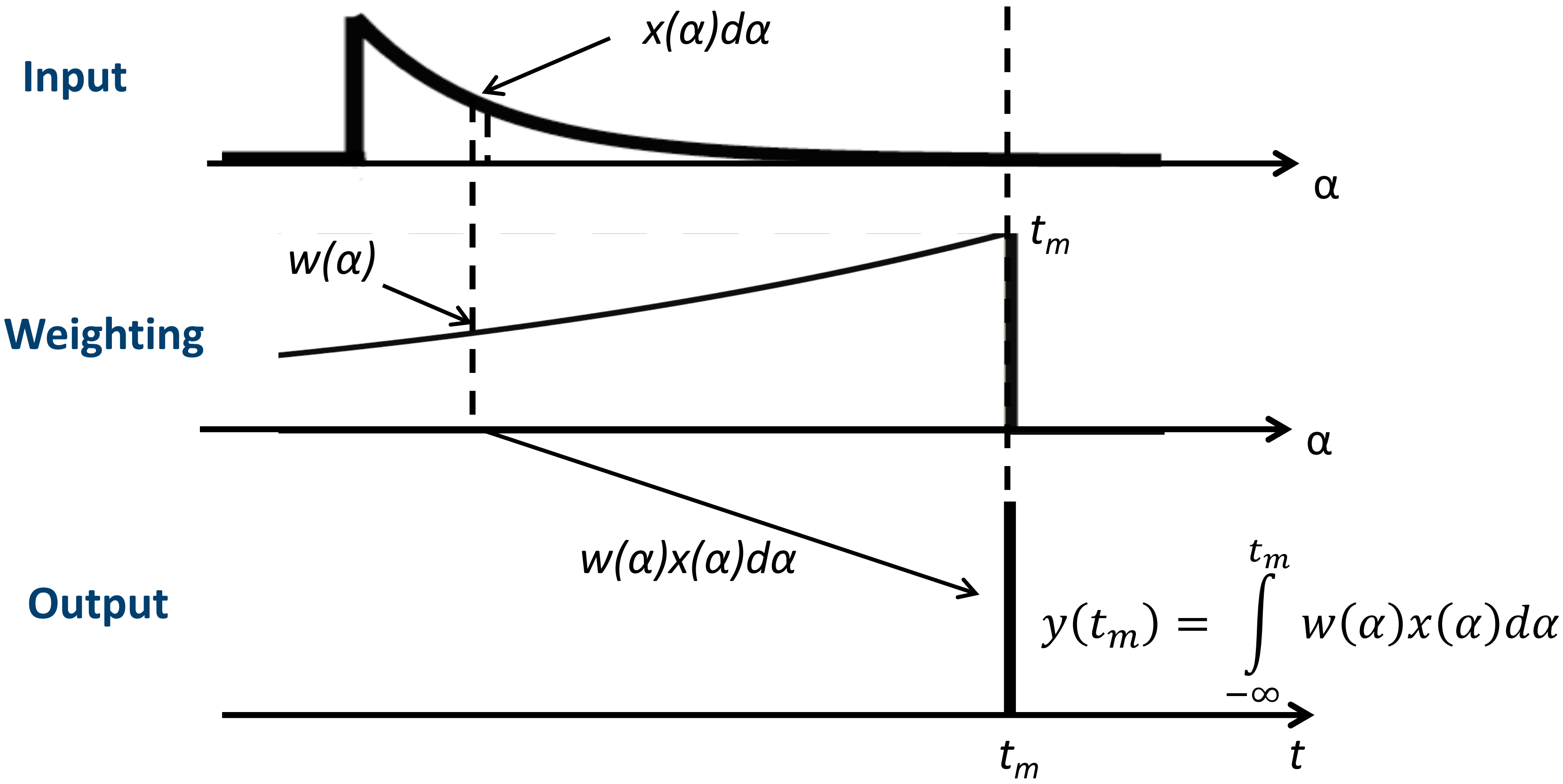
$$w_k x_k$$

$$\boldsymbol{y(t_m) = \sum_{k=0}^{n} w_k x_k}$$

we have a sum of **continuous values**

$$w(\alpha)x(\alpha)$$

$$\boldsymbol{y(t_m) = \int_{-\infty}^{t_m} w(\alpha)x(\alpha)d\alpha}$$



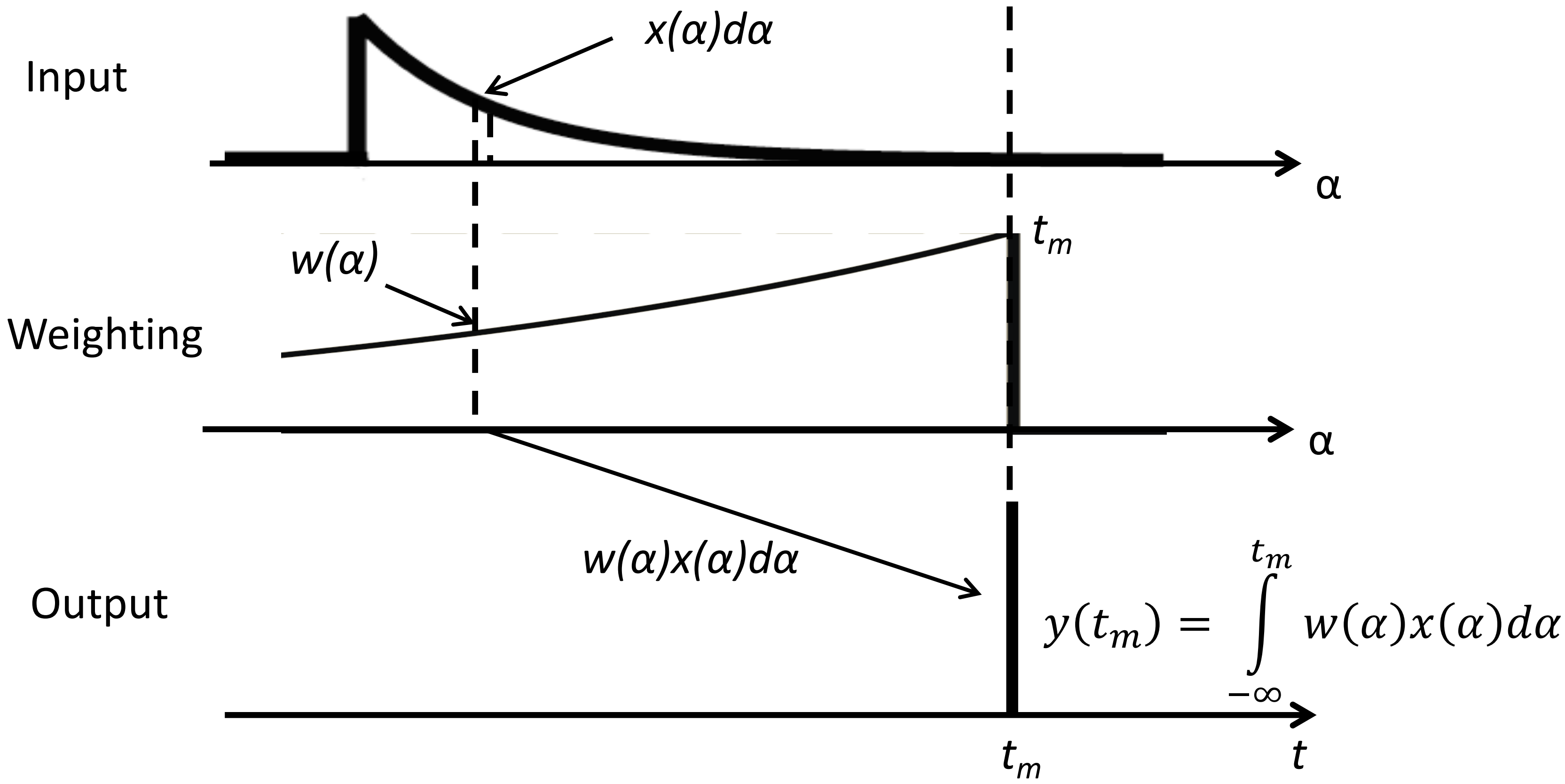
Weighting Function
or
Memory Function

Weighting Function of the Filter

A **weighting function** is defined, which for every element *x(α)dα* of the input denotes the weight *w(α)* given by the filter

w(α) is also called **memory function** of the filter

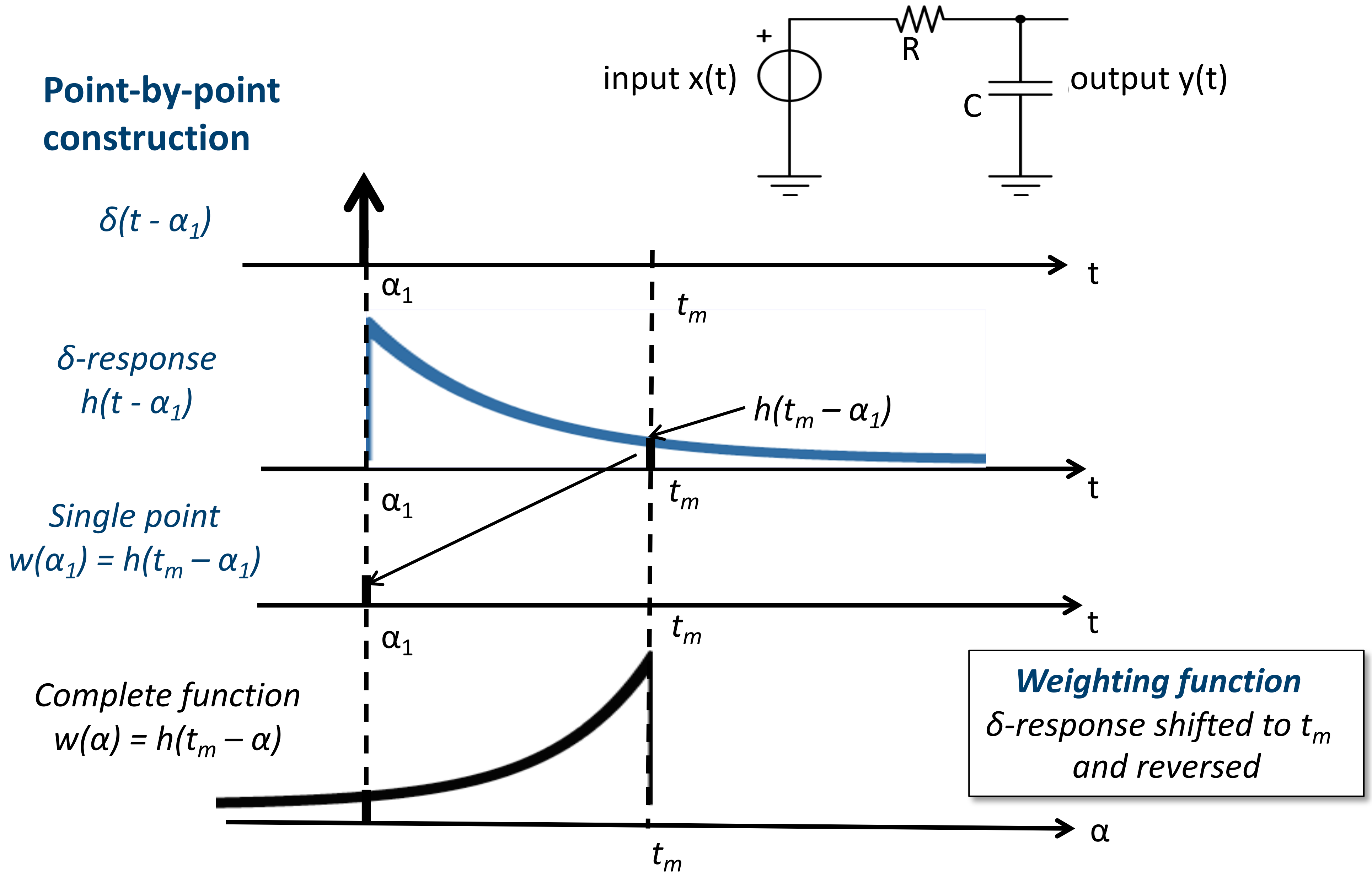
$$y(t_m) = \int_{-\infty}^{t_m} w(\alpha)x(\alpha)d\alpha$$



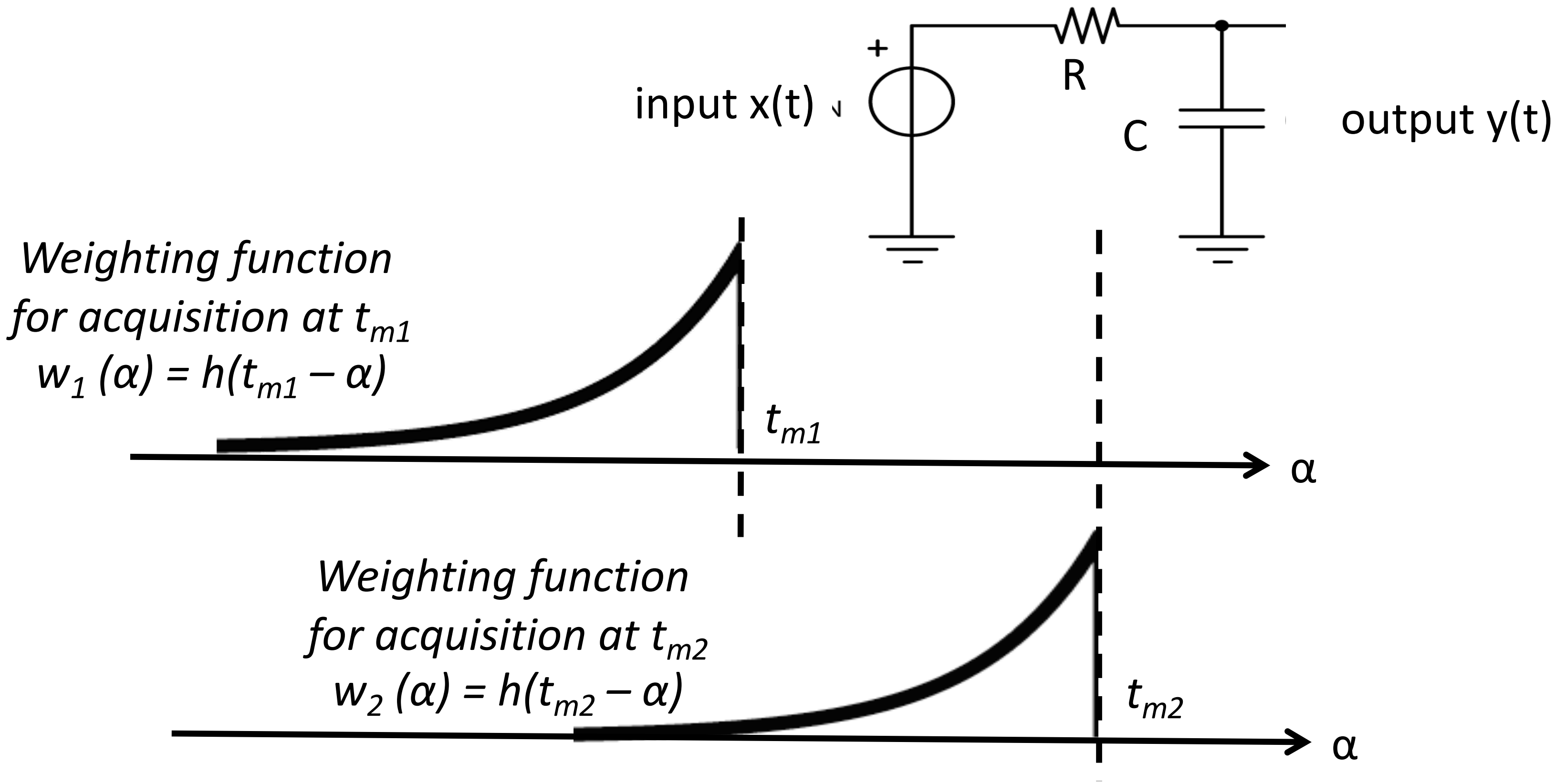
Constant-Parameter Linear Filters

Weighting Function of CONSTANT-PARAMETER Filters

Point-by-point construction



Weighting Function of CONSTANT-PARAMETER Filters



For **constant-parameter** filters the **weighting** function for any t_m has

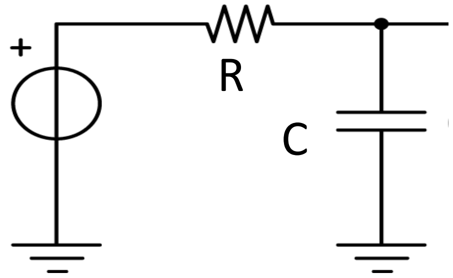
a) always the same shape and

b) always the same time position **with respect to t_m** .

In other words, when $\mathbf{t_m}$ is changed $w(\alpha)$ changes in a very simple way :

"it walks with $\mathbf{t_m}$ just like a tethered dog follows his boss"

Weighting Function of CONSTANT-PARAMETER Filters



We have seen that for constant-parameter filters (*but NOT for time-variant filters !*)

$$w(\alpha) = h(t_m - \alpha)$$

This conclusion is confirmed analytically. The weighting function *w(α)* is definied by

$$y(t_m) = \int_{-\infty}^{t_m} x(\alpha) w(\alpha) d\alpha$$

But for constant-parameter filters (*and NOT for time-variant filters !*)

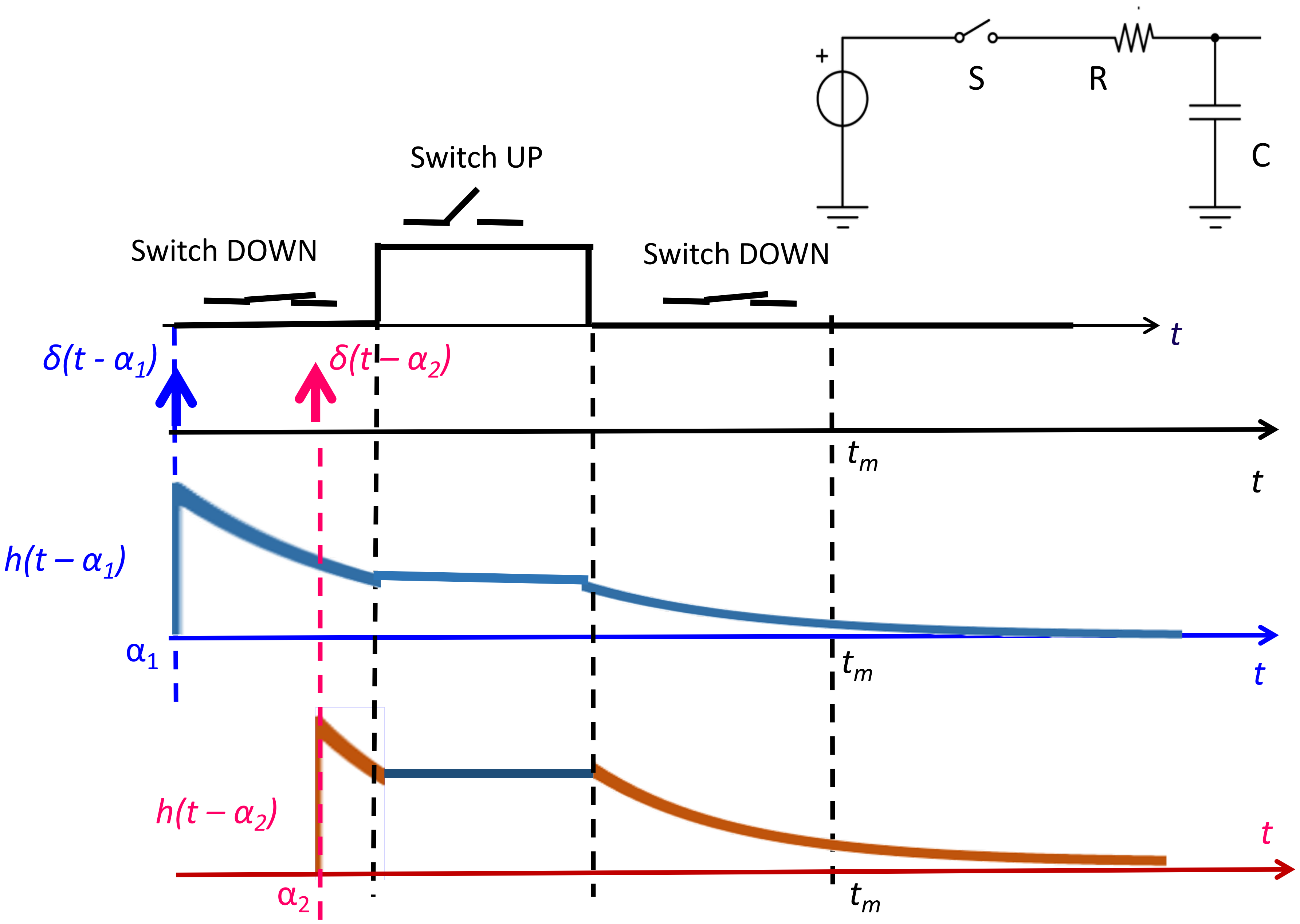
$$y(t_m) = x(t) * h(t) = \int_{-\infty}^{t_m} x(\alpha) h(t_m - \alpha) d\alpha$$

The equations above are both valid for any acquisition time t_m , therefore it is

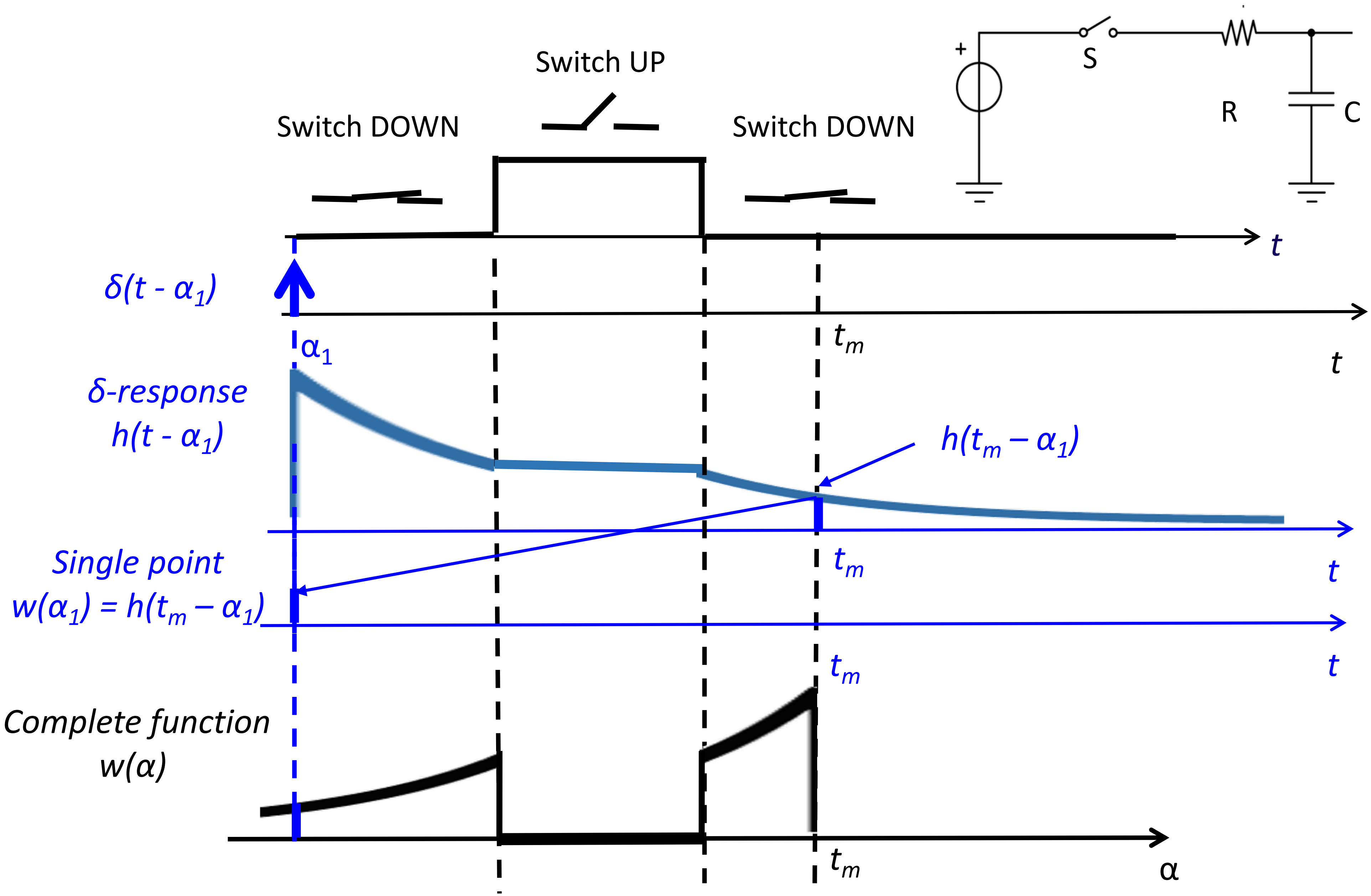
$$\boxed{w(\alpha) \equiv h(t_m - \alpha)}$$

Time-Variant Linear Filters

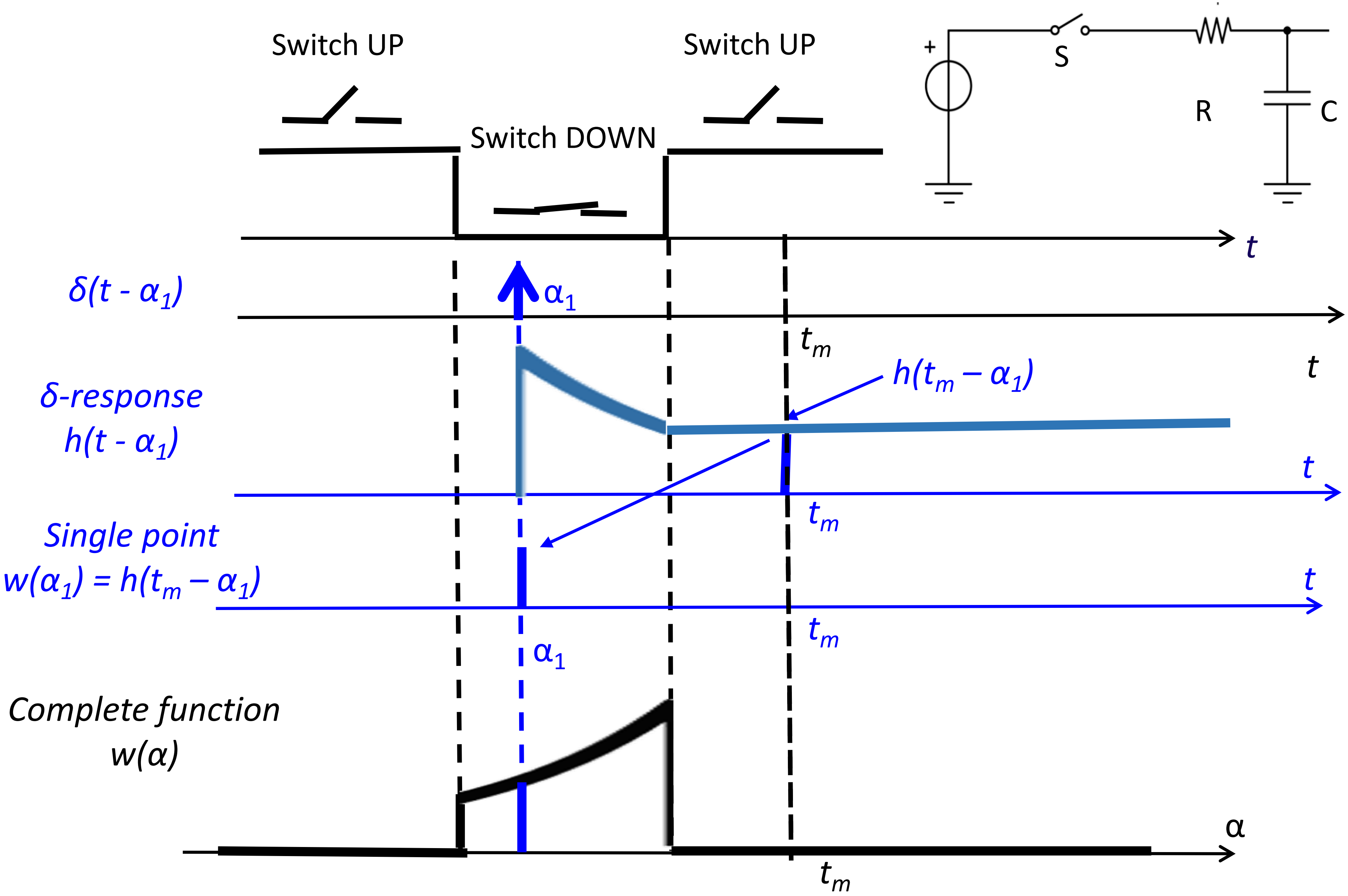
Time-Variant Filters = Variant δ-response



Time-Variant Filters = Variant Weighting



Time-Variant Filters = Variant Weighting



Weighting Function
in the Frequency-Domain

Weighting in the Frequency Domain

The concept of acquired value y(t) as a weighted sum of components can be extended to the frequency domain. Parseval's theorem

$$\int_{-\infty}^{\infty} a^2(t)dt = \int_{-\infty}^{\infty} A(f)A^*(f)df = \int_{-\infty}^{\infty} A(f)A(-f)df$$

can be extended to the product of two functions *a(t)* and *b(t)*

$$\int_{-\infty}^{\infty} a(t)b(t)dt = \int_{-\infty}^{\infty} A(f)B^*(f)df = \int_{-\infty}^{\infty} A(f)B(-f)df$$

Denoting by $W(f) = F[w(t)]$ we have

$$\boldsymbol{y(t_m) = \int_{-\infty}^{\infty} x(\alpha) \cdot w(\alpha)d\alpha = \int_{-\infty}^{\infty} X(f) \cdot W(-f)df}$$

The value y acquired at time tm at the filtering system output can be considered

- either as a weighted sum of instantaneous input values x(t) with weights w(t)
- or as a weighted sum of Fourier components X(f) of the input signal x(t) with weights W(-f) = F[w(-t)]

Weighting Functions: a summary

about weighting functions

- For **constant-parameter** filters (and only for them!) the weighting function is simply the **δ-response function reversed and shifted** in time.
- That's NOT true for **time-variant** linear filters, which **do not have a unique δ-response**. The shape of the δ-response depends on when the δ-function is applied to the input during the evolution in time of the filter.
- The weighting function in linear time-variant filters may be difficult to compute, but it always exists.
- For filters that vary in time with simple law it is fairly simple to compute the weighting function, in particular for switched-parameter filters (see previous examples).
- Switched-parameter filters undergo abrupt changes at the transition from a time interval to the next one, but within each interval the parameters stay constant.
- The values of electrical variables (voltages, currents) before and after the switching must be carefully checked because they can be discontinuous, i.e. they may exhibit abrupt variations at the switching time.