COURSE OUTLINE

- Introduction
- Signals and **Noise: 2) Types and Sources**
- Filtering
- Sensors and associated electronics

Noise Sources and Types

- Shot Noise (or Shottky Noise) main features
- Shot Noise Mean, Mean Square and Power
- Shot Noise Power Spectrum and Autocorrelation Function
- Noise in diodes (Schottky Noise)
- Modeling any Noise with a Poisson Process
- Noise in Resistors (Johnson-Nyquist Noise)
- White Noise

Shot Noise (or Schottky Noise) Main Features

Shot Noise (or Schottky Noise)

Real case: Shot current in a diode

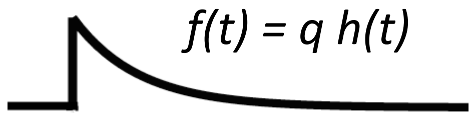
- **Random** sequence of many **independent** pulses,
  i.e. «shots» due to single electrons that swiftly cross the junction depletion layer
- Pulses have rate p, charge q and very short duration T_h
  (shorter than transition times in the circuits)
- «Shot» current has mean value $\boxed{I = p \cdot q}$
- Shot current has fast fluctuations around the mean, called **shot noise**
  (or Schottky noise, after the name of the scientist who explained it)
  The technical literature reports that shot noise has constant spectral density

$$\boxed{\boldsymbol{S_{nu} = 2qI}}$$ **unilateral density** in $0 < f < \infty$

Let us see how this result can be inferred from the basic features of the shot process

Shot Current

Current in a reverse-biased **p-n junction diode**
random sequence of **independent** elementary pulses *f(t)*
(single carriers that fall down the potential barrier and cross the depletion layer)

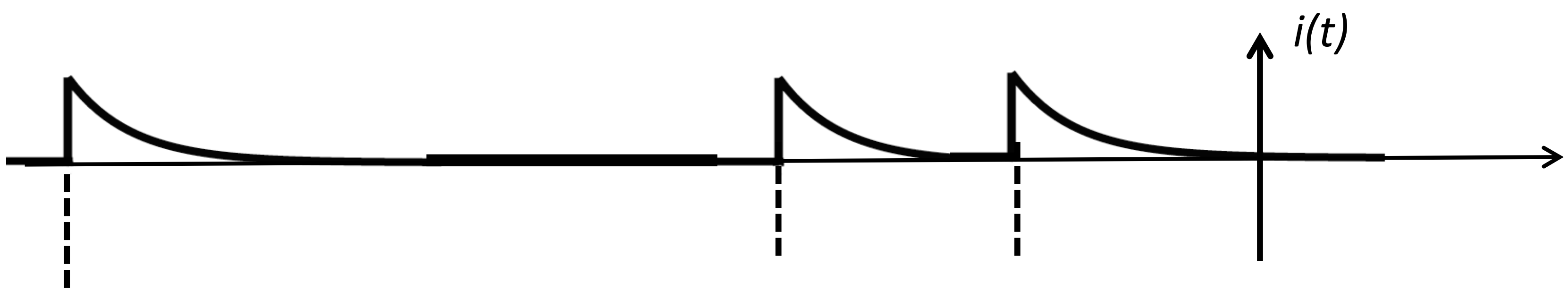


q pulse charge
h(t) normalized pulse shape: $\int_{-\infty}^{\infty} h(t)dt = 1$

POISSON STATISTICAL PROCESS

- The pulses are **independent statistical** events:
  the probability for a pulse to occur is independent of the occurrence of other pulses
- $p \cdot dt$ is the probability that a pulse starts in $t \longleftrightarrow t+dt$
- We consider *p* constant (independent of t)

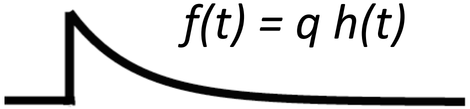
at time t the current *i* is the superposition of the contributions of pulses starting before t



Shot Noise
Mean, Mean Square and Power

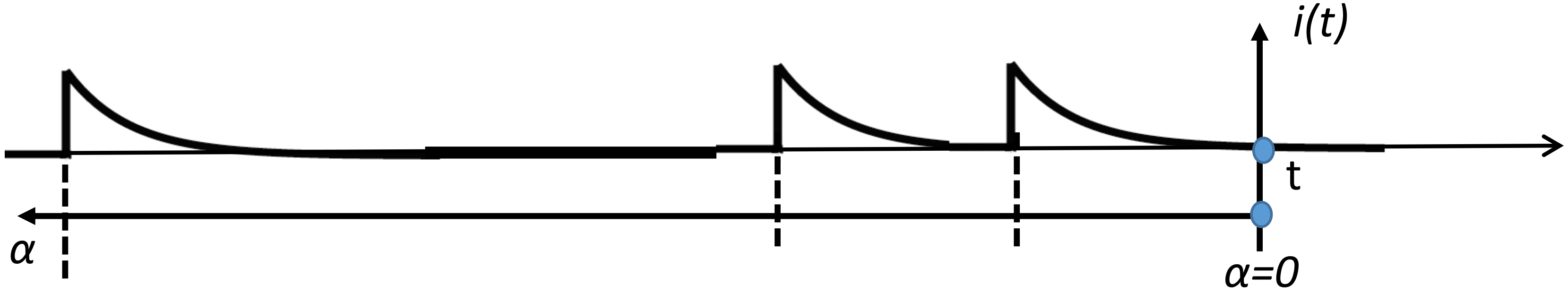
Shot Current: Mean Value

Shot current = **random** sequence of **independent** elementary pulses *f(t)*



q pulse charge

$h(t)$ pulse shape (normalized $\int_{-\infty}^{\infty} h(t)dt = 1$)



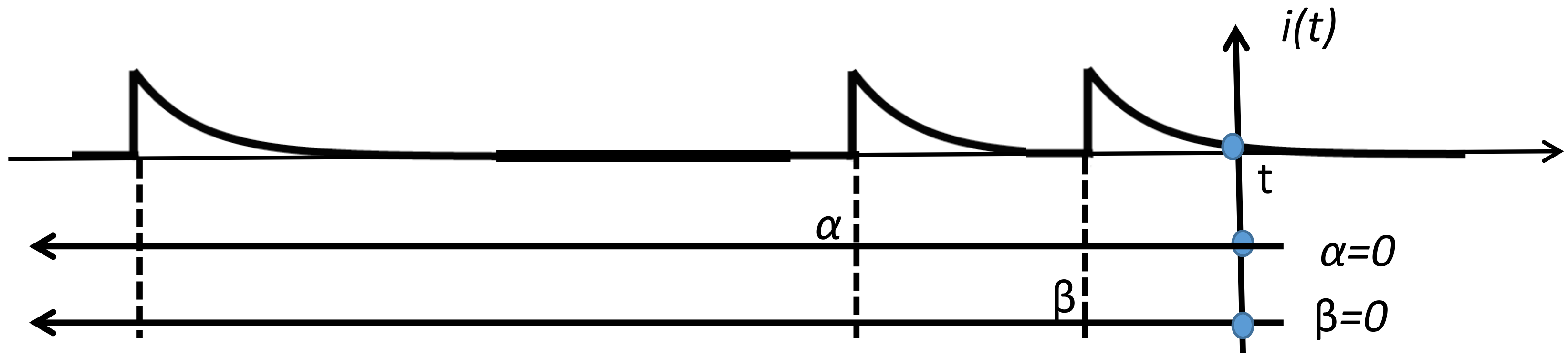
- A pulse which starts at time α contributes a current $q\,h(\alpha)$ at time t
- $p d\alpha$ probability that a pulse starts in $\alpha \longleftrightarrow \alpha + d\alpha$

Mean current at time t : **sum of the mean effects** of all possible pulses

$$\overline{i(t)} = I = \int_0^{\infty} q h(\alpha) \cdot p d\alpha = pq \int_0^{\infty} h(\alpha) d\alpha = pq$$

Evident without computation: current = pulses at rate p each one carrying charge q.
However, with this approach to computation we can get also the **mean square** value

Shot Current: Mean Square



Let's consider a couple of pulses starting one at time α and the other at time β

- The contribution of the couple to the square $i^2(t)$ of the current at time t is $[qh(\alpha) + qh(\beta)]^2 = q^2 h^2(\alpha) + q^2 h^2(\beta) + qh(\alpha) \cdot q\, h(\beta) + q\, h(\beta) \cdot qh(\alpha)$
- i.e. a pulse gives two contributions, one of its own («square» terms $q^2 h^2(\alpha)$) plus one in collaboration with the other pulse («rectangular» terms $q\, h(\alpha) \cdot q\, h(\beta)$)
- The probability that a «square» contribution $q^2 h^2(\alpha)$ exists is **SIMPLY** the probability that the pulse in α exists, i.e. $p d\alpha$
- The probability that a «rectangular» contribution $qh(\alpha) \cdot qh(\beta)$ exists is the probability that the pulse in α exists **AND** the pulse in β exists, i.e. $p d\alpha \cdot p d\beta$
- The mean contribution given in total by the couple is therefore

$$\mathbf{d}\boldsymbol{i^2} = \boldsymbol{q^2 h^2(\alpha) p d\alpha + q^2 h^2(\beta) p d\beta + qh(\alpha) p d\alpha \cdot q\, h(\beta) p d\beta + q\, h(\beta) p d\beta \cdot qh(\alpha) p d\alpha}$$

Shot Current: Mean Square and Noise Power

The mean square value $\overline{i^2(t)}$ of the current is given by the sum of the mean contributions of all possible single pulses and of all possible couples of pulses

$$\overline{i^2(t)} = \int_0^\infty q^2 h^2(\alpha) \cdot p d\alpha + \iint_0^\infty q h(\alpha)\, p d\alpha\, q h(\beta)\, p\, d\beta =$$

$$= pq^2 \int_0^\infty h^2(\alpha) d\alpha + pq \int_0^\infty h(\alpha) d\alpha \cdot pq \int_0^\infty h(\alpha) d\alpha =$$

$$= pq^2 \int_0^\infty h^2(\alpha) d\alpha + (pq)^2 = pq^2 \int_0^\infty h^2(\alpha) d\alpha + \left(\overline{i(t)}\right)^2 =$$

$$= pq^2 \int_0^\infty h^2(\alpha) d\alpha + I^2$$

The current **noise** is only the **fluctuation** of *i(t)* around the mean value *I*

$$n_i(t) = i(t) - \overline{i(t)} = i(t) - I$$

hence the noise power is the mean square **deviation** of i(t) :

$$\overline{n_i{}^2} = \overline{i^2(t)} - \left(\overline{i(t)}\right)^2 = pq^2 \int_0^\infty h^2(\alpha) d\alpha =$$

$$\boldsymbol{\overline{n_i{}^2} = qI \int_0^\infty h^2(\alpha) d\alpha}$$

This is the **Campbell Theorem**

Shot Noise Power Spectrum
and
Autocorrelation Function

Shot Noise: Power Spectrum

$$\overline{n_i^{\,2}} = qI\int_0^{\infty} h^2(\alpha)d\alpha = qI\int_{-\infty}^{\infty} h^2(\alpha)d\alpha$$

NB: $-\infty$ because $h(\alpha) = 0$ for $\alpha<0$

- Let's denote by *H(f)* the Fourier transform of the elementary pulse shape *h(t)*

$$H(f) = F[h(t)]$$

- By the *Parseval theorem*, we see that the noise power is a sum of contributions of elementary components in frequency domain

$$\overline{n_i^{\,2}} = qI\int_{-\infty}^{\infty} h^2(\alpha)d\alpha = qI\int_{-\infty}^{\infty} |H(f)|^2\, df$$

- The noise power computed from the power spectrum $S_n(f)$ is

$$\overline{n_i^{\,2}} = \int_{-\infty}^{\infty} S_n(f)df$$

- Hence the power spectrum is

$$\boldsymbol{S_n(f) = qI\,|H(f)|^2}$$

Shot Noise: Autocorrelation Function

From the shot noise power spectrum

$$S_n(f) = qI|H(f)|^2$$

$$R_{nn}(\tau) = F^{-1}[S_n(f)] = qI \cdot F^{-1}[|H[f]|^2] = qI \cdot k_{hh}(\tau)$$

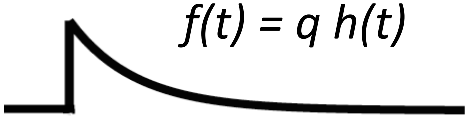
we obtain the shot noise autocorrelation

$$n_i^2 = R_{nn}(0) = qI \cdot k_{hh}(0) = qI \cdot \int_0^{\infty} h^2(\alpha)\ \ d\alpha$$

The shot noise power can be computed also in the time domain

Shot Noise: summary

Shot current *i(t)* : **random** sequence of **independent** elementary pulses *f(t)* with probability density *p* in time



q pulse charge

h(t) normalized pulse shape $\int_{-\infty}^{\infty} h(t)dt = 1$

Shot noise $n_i(t)$: **random fluctuations** of the current **around its mean value** *I = pq*

TIME DOMAIN

Autocorrelation

$$R_{nn}(\tau) = pq^2 \cdot k_{hh}(\tau) =$$

$$= qI \int_0^{\infty} h(\alpha)h(\alpha+\tau)\, d\alpha$$

Mean square value

$$\overline{n_i{}^2} = pq^2 \cdot k_{hh}(0) = qI \int_0^{\infty} h^2(\alpha)d\alpha$$

FREQUENCY DOMAIN

Power Spectrum

$$S_n(f) = pq^2 \cdot |H(f)|^2 =$$

$$= qI \cdot |H(f)|^2$$

Mean square value

$$\overline{n_i{}^2} = \int_{-\infty}^{\infty} S_n(f)df$$

Noise in Diodes

Diode Noise in Reverse Bias

In a p-n reverse-biased diode with mean current I_S we have:

- **Elementary pulses:** currents induced at the terminals by single carriers that fall down the potential barrier and cross the junction depletion layer
- **Elementary pulse width T_h** = transit time in the junction (from *a few ps* to *1ns* , since the p-n depletion layer ranges from 0,1 *μm* to *100 μm*)
- We can assume

  $$h(t) \cong \delta(t) \qquad \text{and} \qquad |H(f)| \cong 1$$

  with approximation valid for correlation times longer than T_h (i.e. down to ns) that is, for frequencies up to ≈ $1/T_h$ (i.e. up to GHz)

- With this approximation the noise spectrum is

  $$S_{nB}(f) = qIS \cdot |H(f)|^2 \cong qI_S \qquad (S_{nB} \text{ \underline{bilateral} density})$$

  $$\boldsymbol{S_{nU}(f) = 2qI_S \cdot |H(f)|^2 \cong 2qI_S} \qquad (S_{nU} \text{ \underline{unilateral} density})$$

  which is just the equation reported in the literature

- The corresponding noise autocorrelation is δ-like

  $$R_{nn}(\tau) = qIS \cdot k_{hh}(\tau) \cong qIS \cdot \delta(t)$$

Diode Noise in forward bias

$$I = I_S\left(e^{\frac{qV}{kT}} - 1\right) = I_S e^{\frac{qV}{kT}} - I_S$$

The diode current is the result of opposite shot components with mean values:

a) $-I_S$ reverse current of minority carriers, which fall down the potential barrier

b) $I_S\, e^{\frac{qV}{kT}}$ forward current of majority carriers,which jump over the potential barrier

- The **mean** current is the **difference** of the components
- The independent current **fluctuations are quadratically added** in the spectrum

$$S_{nU}(f) = 2qI_S e^{\frac{qV}{kT}} + 2qI_S = 2q(I + I_S) + 2qI_S = 2qI + 4qI_S$$

- In forward bias it is $I \gg I_S$ and the spectrum is

$$\boxed{S_{nU}(f) \approx 2qI}$$

- At zero bias it is I=0 and the spectrum is

$$\boxed{S_{nU}(f) \approx 4qI_S}$$

Noise in Resistors (Johnson-Nyquist noise)

Resistor thermal noise (Johnson-Nyquist noise)

- **The voltage V between the terminals of a conductor with resistance R shows random fluctuations that do not depend on the current *I***
- The technical literature reports that this noise has voltage spectral density S_{vU} constant up to very high frequency >> 1GHz: denoting by R the resistance and by T the absolute temperature it is

$$\boldsymbol{S_{vU}(f) = 2kTR} \quad \textbf{(bilateral)}$$

- This noise can be described also in terms of current in the conductor terminals: denoting by $G = 1/R$ the conductance, the current spectral density is

$$\boldsymbol{S_{iU}(f) = 2kTG} \quad \textbf{(bilateral)}$$

- This noise is known as ***Johnson-Nyquist noise***, after the name of the scientists that first studied and explained it.
- It is generated by the agitation of the charge carriers (electrons) in the conductor in thermal equilibrium at temperature T

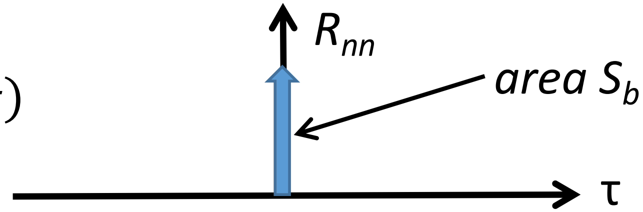
White Noise

White Noise (stationary)

The **IDEAL «white» noise** is a concept extrapolated from Johnson noise and shot noise and is defined by the **essential** characteristic feature:

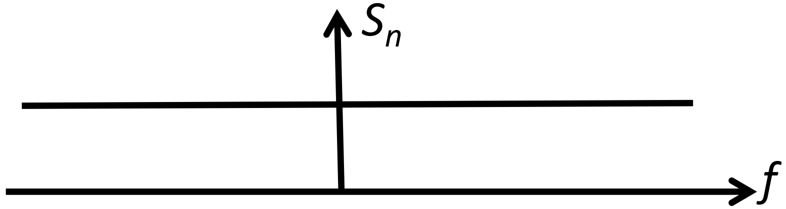
no autocorrelation at any time distance τ, no matter how small

$$R_{nn}(\tau) = S_b \cdot \delta(\tau)$$



and therefore constant spectrum

$$S_n(f) = S_b$$



In reality such a noise does not exist: it would have divergent power $\overline{n^2} \rightarrow \infty$

A **REAL «white»** noise has

- **Very small width** of autocorrelation, shorter than the minimum time interval of interest in the actual case and therefore approximated to zero
- **Very wide band** with constant spectral density S_b , wider than the maximum frequency of interest in the actual case and therefore approximated to infinite

White Noise (non-stationary)

Also in non-stationary cases the IDEAL «white» noise *n(t)* is defined by the **essential** characteristic feature:

no correlation at any finite time distance τ, no matter how small,

but the noise intensity is no more constant, it varies with time *t*

that is

the autocorrelation function is δ-like,

but has time-dependent area $S_b(t)$

$$R_{nn}(t, t+\tau) = S_b(t) \cdot \delta(\tau)$$