COURSE OUTLINE

- Introduction
- Signals and **Noise: 2) Types and Sources**
- Filtering
- Sensors and associated electronics

Noise Analysis and Simulation

- Noise in diodes (Schottky Noise)
- Noise in Resistors (Johnson-Nyquist Noise)
- White Noise
- Band-Limited White Noise (Wide-Band Noise)
- Basic Parameters of Wide-Band Noise

Noise in Diodes

Shot Noise (or Schottky Noise)

Real case: Shot current in a diode

- **Random** sequence of many **independent** pulses,
  i.e. «shots» due to single electrons that swiftly cross the junction depletion layer
- Pulses have rate p, charge q and very short duration T_h
  (shorter than transition times in the circuits)
- «Shot» current has mean value $I = p \cdot q$
- Shot current has fast fluctuations around the mean, called **shot noise**
  (or Schottky noise, after the name of the scientist who explained it)
  The technical literature reports that shot noise has constant spectral density

$$S_{nu} = 2qI$$

unilateral density in $0 < f < \infty$

Diode Noise in forward bias

$$I = I_S\left(e^{\frac{qV}{kT}} - 1\right) = I_S e^{\frac{qV}{kT}} - I_S$$

The diode current is the result of opposite shot components with mean values:

a) $-I_S$ reverse current of minority carriers, which fall down the potential barrier

$b)$ $I_S\, e^{\frac{qV}{kT}}$ forward current of majority carriers,which jump over the potential barrier

- The **mean** current is the **difference** of the components
- The independent current **fluctuations are quadratically added** in the spectrum

$$S_{nU}(f) = 2qI_S e^{\frac{qV}{kT}} + 2qI_S = 2q(I + I_S) + 2qI_S = 2qI + 4qI_S$$

- In forward bias it is $I \gg I_S$ and the spectrum is

$$\boxed{S_{nU}(f) \approx 2qI}$$

- At zero bias it is I=0 and the spectrum is

$$\boxed{S_{nU}(f) \approx 4qI_S}$$

Noise in Resistors
(Johnson-Nyquist noise)

Resistor thermal noise (Johnson-Nyquist noise)

- **The voltage V between the terminals of a conductor with resistance R shows random fluctuations that do not depend on the current *I***
- The technical literature reports that this noise has voltage spectral density S_{vU} constant up to very high frequency >> 1GHz: denoting by R the resistance and by T the absolute temperature it is

$$S_{vU}(f) = 2kTR \quad \textbf{(bilateral)}$$

- This noise can be described also in terms of current in the conductor terminals: denoting by $G = 1/R$ the conductance, the current spectral density is

$$S_{iU}(f) = 2kTG \quad \textbf{(bilateral)}$$

- This noise is known as ***Johnson-Nyquist noise***, after the name of the scientists that first studied and explained it.
- It is generated by the agitation of the charge carriers (electrons) in the conductor in thermal equilibrium at temperature T

White Noise

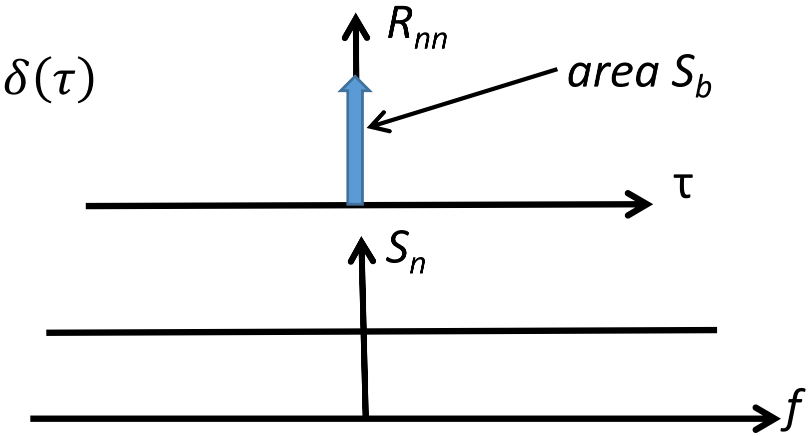
White Noise (stationary)

IDEAL «white» noise
is a concept extrapolated from Johnson noise and shot noise
defined by its **essential** feature:
no autocorrelation at any time distance τ, no matter how small

$$R_{nn}(\tau) = S_b \cdot \delta(\tau)$$

and therefore constant spectrum

$$S_n(f) = S_b$$



In reality such a noise does not exist: it would have divergent power $\overline{n^2} \to \infty$

REAL «white» noise has

- **Very small width** of autocorrelation, shorter than the minimum time interval of interest in the actual case and therefore approximated to zero
- **Very wide band** with constant spectral density S_b , wider than the maximum frequency of interest in the actual case and therefore approximated to infinite

White Noise (non-stationary)

Also in non-stationary cases the IDEAL «white» noise
is defined by the **essential** characteristic feature:

no correlation at any finite time distance τ, no matter how small,

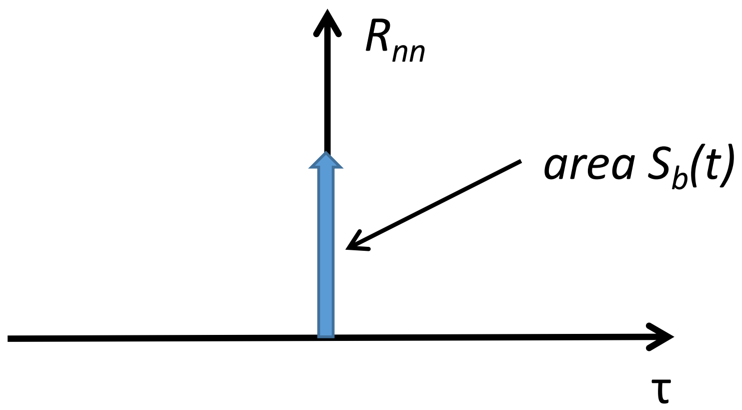
but the noise intensity is no more constant, it varies with time *t*

that is

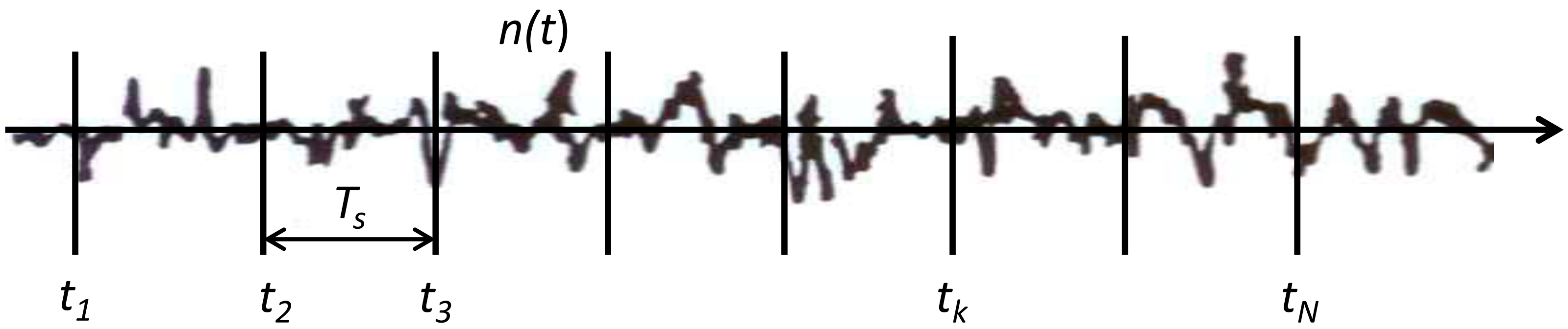
the autocorrelation function is δ-like,

but has **time-dependent area $S_b(t)$**

$$R_{nn}(t, t+\tau) = S_b(t) \cdot \delta(\tau)$$



Filtering white noise is simple



For clarity, let’s consider a discrete case:

linear filtering in digital signal processing:

- Sample n_1 at t_1 and multiply by a weight w_1 ,
- sample n_2 at $t_2 = t_1 + T_s$ and multiply by a weight w_2 and sum
- and so on ....

The filtered noise n_f is

$$n_f = w_1 n_1 + w_2 n_2 + \; \ldots\ldots = \sum_{k=1}^{N} w_k n_k$$

and its mean square value is

$$\overline{n_f{}^2} = \overline{w_1{}^2 n_1{}^2 + w_2{}^2 n_2{}^2 + \ \ldots + w_1 n_1 \cdot w_2 n_2 + w_1 n_1 \cdot w_3 n_3 + \cdots} =$$

$$= w_1{}^2 \overline{n_1{}^2} + w_2{}^2 \overline{n_2{}^2} + \ \ldots\ w_1\, w_2 \overline{n_1 n_2} + w_1 w_3 \overline{n_1 n_3} + \cdots$$

Filtering white noise is simple

$$\overline{n_f^{\,2}} = \overline{w_1^{\,2} n_1^{\,2} + w_2^{\,2} n_2^{\,2} + \ \ldots + w_1 n_1 \cdot w_2 n_2 + w_1 n_1 \cdot w_3 n_3 + \cdots} =$$

$$= w_1^{\,2}\overline{n_1^{\,2}} + w_2^{\,2}\overline{n_2^{\,2}} + \ \ldots\ w_1\, w_2 \overline{n_1 n_2} + w_1 w_3 \overline{n_1 n_3} + \ldots$$

If noise at interval T_s is **not correlated**, then all rectangular terms vanish

$$\overline{n_1 n_2} = \overline{n_1 n_2} = \cdots = 0$$

and the result is **simply a sum of squares**, even in case of non-stationary noise

$$\overline{n_f^{\,2}} = w_1^{\,2}\overline{n_1^{\,2}} + w_2^{\,2}\overline{n_2^{\,2}} + \ \ldots =$$

$$= \sum_{k=1}^{N} w_k^{\,2}\overline{n_k^{\,2}}$$

If the noise is stationary

$$\overline{n_1^{\,2}} = \overline{n_2^{\,2}} = \overline{n_3^{\,2}} = \cdots = \overline{n^2}$$

there is a further simplification

$$\overline{n_f^{\,2}} = \overline{n^2}\ (w_1^{\,2} + w_2^{\,2} + \ \ldots) =$$

$$= \overline{n^2}\ \sum_{k=1}^{N} w_k^{\,2}$$

we will see later that also with continuous filtering white noise brings similar simplification

Band-Limited White Noise

or

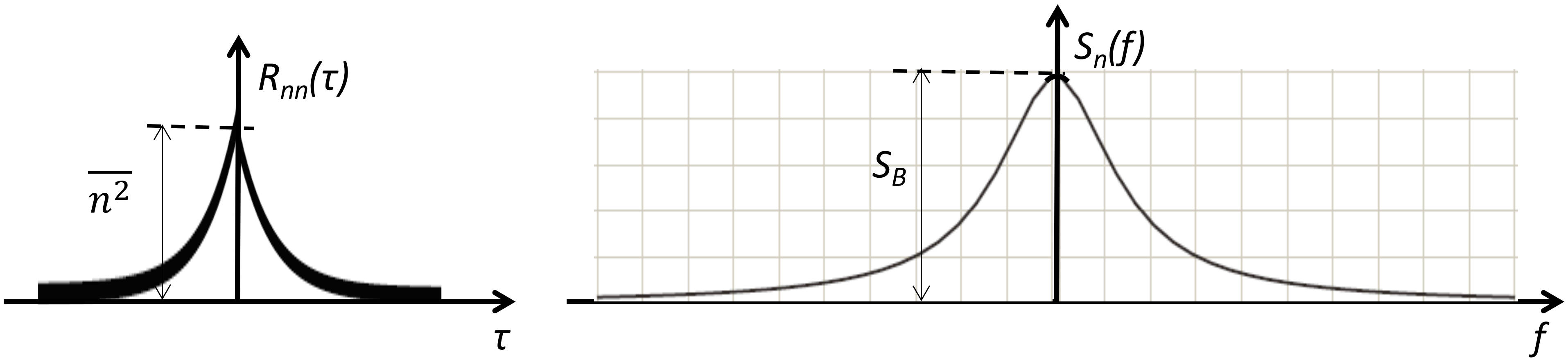
Wide-Band Noise

Band-limited white noise (wide-band noise)

- **Real white noise** = white noise with band limited at high frequency
- *The limit may be inherent in the noise source or due to low-pass filtering enforced by the circuitry*. **Anyway, in all real cases there is such a limit**
- A frequent typical case is the **Lorentzian** spectrum:
  band limited by a **simple pole** with time constant T_p , pole frequency $f_p = 1/2\pi T_p$

$$R_{nn}(\tau) = \overline{n^2} e^{-\frac{|\tau|}{T_p}}$$

$$S_n(f) = \frac{S_B}{1 + \left(2\pi f T_p\right)^2}$$

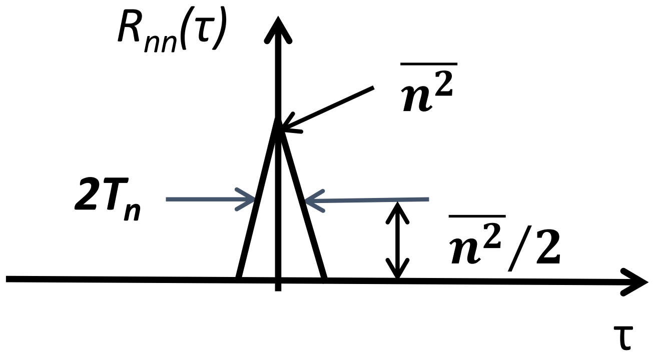


Basic Parameters of Wide-Band Noise

Simplified description of wide-band noise

The true $R_{nn}(\tau)$ and $S_n(f)$ can be **approximated retaining the noise main features**:

a) equal **mean square $\overline{n^2}$** and b) equal spectral **density S_b**



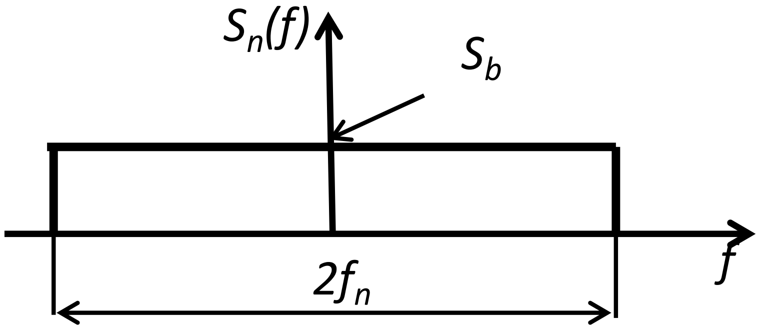
in time: $R_{nn}(\tau)$ triangular approx, half-width $2T_n$

a) **equal msq noise** : $R_{nn}(0) = \overline{n^2}$

b) **equal spectral density**: [area of $R_{nn}(\tau)$] = S_b ,

(i.e. $\overline{n^2}\,2\,T_n = S_b$)

Correlation width = $\Delta\tau = 2T_n$



in frequency: $S_n(f)$ rectang approx, half-width f_n

a) **equal msq noise** : [area of $S_n(f)$] = $\overline{n^2}$

(i.e. $S_b 2\, f_n = \overline{n^2}$)

b) **equal spectral density**: $S_n(0) = S_b$

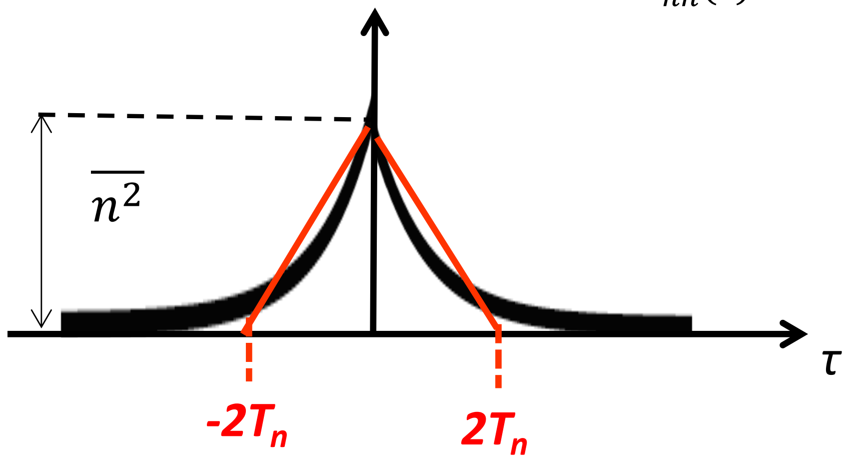
Noise bandwidth: $\Delta f = 2f_n$

Note that $\Delta\tau \cdot \Delta f = 1$ which is consistent with $S_n(f) = F[R_{nn}(\tau)]$

Simplified description of Lorentzian spectrum

Time

$$R_{nn}(\tau) = \overline{n^2} e^{-\frac{|\tau|}{T_p}}$$

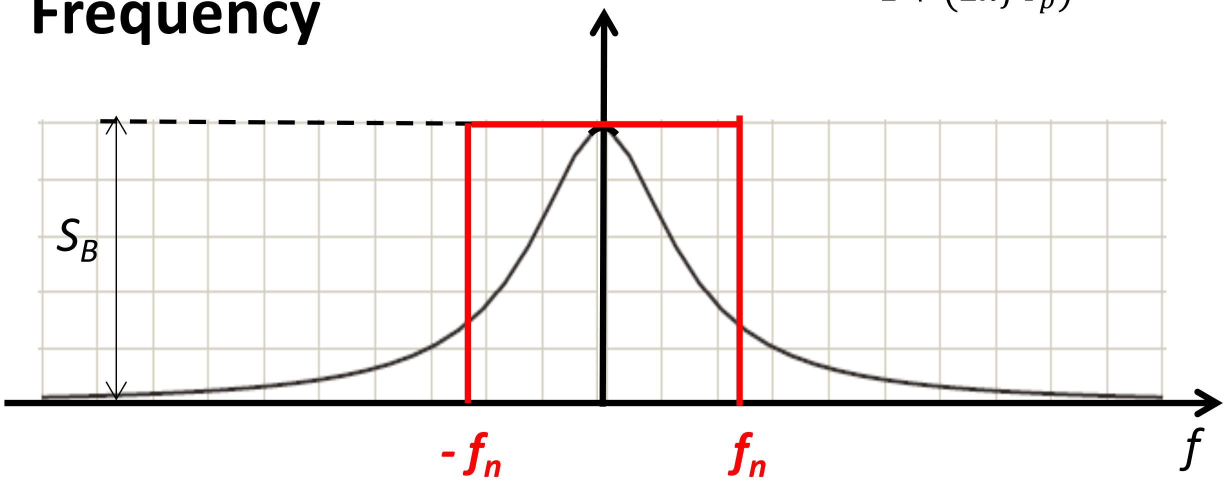


$$S_B = \int_{-\infty}^{\infty} R_{nn}(\tau)\ d\tau = \overline{n^2}\ 2T_p$$

$$T_n = T_p$$

Frequency

$$S_n(f) = \frac{S_B}{1 + (2\pi f T_p)^2}$$



$$\overline{n^2} = \int_{-\infty}^{\infty} S_n(f)\ df = S_B \frac{1}{2T_p}$$

$$2f_n = \frac{1}{2T_p}$$

Note that $f_n \neq f_p$, namely $f_n = \frac{1}{4T_p} = \frac{\pi}{2} f_p$